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Numerical analysis methods applied to reserve estimates of stacked granular solids.

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NUMERICAL ANALYSIS METHODS APPLIED TO RESERVE ESTIMATES OF STACKED GRANULAR SOLIDS

by

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CHAPTER I

INTRODUCTION

Business men the world over, have long recognized that records and reports of the physical quantities and costs of inventories, including their balances, changes and relationships, are essential to managerial appraisal of past business performance and to planning and control of future operations.[1] The objective of inventory management is to maintain the investment in inventories at the lowest amount which is sufficient to meet production, sales and financial requirements of the enterprise. The inventories must be adequate to maintain an efficient level of operations and to meet, within reason, the needs of customers. However, it must not be greater than is necessary to meet these requirements because of interest costs, the cost of handling and storing excessive quantities of inventory, the dangers of adverse price changes and obsolescense, and the increased exposure to physical deterioration.[2] Thus the requirement for an accurate inventory determination is a prerequisite to effective inventory management.

For most business enterprises, the determination of inventory assets presents no unusual problems since most inventory assets can be physically ascertained. However, in certain businesses such as the manufacture of chemicals, major portions of their inventory assets are in bulk raw and

finished materials. The problem of accurately determining inventory balances of bulk materials has continually plagued business managers because such materials are not susceptable to recognized mensuration techniques. The quantifying of bulk inventories is especially difficult when the economics of handling and storing the bulk materials dictate that they be stored in stockpiles which have a great degree of surface non-conformity and no constraining boundaries.[3]

Purpose and Scope

The purpose of this thesis is to determine, by experiment, the minimum measurements necessary to estimate the content of irregular solids within a prescribed error using numerical analysis methods. The work encompasses a laboratory study utilizing granular ammonium nitrate for construction of stockpiles having varying configurations. The study and the results of the laboratory tests are to be applied to determine the reserves of ammonium nitrate stored in the bulk storage warehouses of the Cooperative Farm Chemical Association at Lawrence, Kansas.

In the laboratory experimental project, a digital computer program and a manual computation method are developed to estimate the content of stockpiles of granular materials. Additionally, the angle of repose of coated ammonium nitrate granules is ascertained and compared with the accepted value for this characteristic.

Historical Perspective of Volume Calculations

For centuries, mathematicians have known the formulas for calculating the volume of linear and non-linear solids.[4] These formulas require explicit measurements of the length of a line or the size of an angle. Examples of such volumes are spheres, cones, pyramids and solids of revolution such as ellipsoids and cylinders. Volume determinations may also be based on implicit measurements. This field encompasses the science of integral calculus wherein volumes are computed by integration of either a known analytic function or a finite numerical difference expression of the area or one linear dimension as a function of the other two dimensions. However, very little application has been made using the numerical difference approach since the amount of computational work is large by this method. Thus, until the advent of the digital computer, the best course of action was to estimate the volume of irregular solids using the classical explicit volume formulas and to accept the error inherent with this method. The present study is an investigation of the benefits to be gained through the use of numerical integration of various numerical difference expressions of the area or height of a stockpile.

Problems Associated with Stockpile Volume Estimation

Undoubtably the biggest problem in measuring stockpiles is their size. This characteristic coupled with the irregular shape of the stockpile surface precludes normal

measurement methods based on geometrical formulas. Additionally, if the stockpile is composed of granular material it is impractical, if not impossible, to take depth measurements over the surface because of the fluidity of the granules. Consequently, the practice of surveying the surface with a transit came into vogue in an attempt to estimate the content of the stockpile. This procedure is quite laborious and the accuracy of the volume calculations leaves something to be desired.

Aerial photocontouring is feasible for outdoor stockpiles. Results by this procedure are fairly accurate but this method is expensive and takes considerable time due to the limited number of photocontouring firms.

Photocontouring on the other hand is impractical for many small outdoor industrial stockpiles. Needless to say it is nearly impossible to photocontour indoor stockpiles due to factors such as size of the warehouse and environmental conditions such as dust.

Another problem associated with determining the content of stockpiles is the amount of time and cost involved in taking the measurements and performing the necessarily calculations to fix the inventory quantity. In exercising sound inventory management control, engineers and businessmen should be knowledgeable of the alternatives available in terms of inventory accuracy versus the time and costs required to attain that accuracy.

CHAPTER II

NUMERICAL ANALYSIS METHODS

Numerical Integration

Integration may be defined as a process of summation. Numerical integration is defined by J. B. Scarborough "as the process of computing the value of a definite integral from a set of numerical values of the integrand".[5] This process is sometimes called mechanical quadrature when applied to the integration of a function of a single variable; when applied to the calculation of a double integral of a function of two independent variables it is called mechanical cubature.

For numerical integration, the problem is to estimate the numerical value of the integral:

$$I(x) = \int_a^b f(x) \; dx \tag{2.1}$$

when f(x) either is too complicated an analytical function to permit the integration in an analytic manner or when the value of f(x) is given only in tabulated form. One method of solving the problem is to expand f(x) in an infinite series, the individual terms of which can be integrated. Thus if

$$f(x) = \sum_{n=0}^{\infty} A_n \, \phi_n \, (x) \tag{2.2}$$

and one can determine the constants

$$C_n = \int_a^b \phi_n (x)\, dx$$

then

$$I(x) = \int_a^b f(x)\, dx = \sum_{n=0}^{\infty} A_n C_n \tag{2.3}$$

Such a method is practical, provided this series converges rapidly, i.e., only a few terms need to be calculated to determine I(x) to any desired accuracy.

The problem of numerical integration is solved for the general case by representing the integrand by a suitable polynomial, for a given interval and subsequently integrating the polynomial between the desired limits. The polynomials used for this purpose are called interpolating polynomials. They are derived using the methods of finite differences. Finite difference methods are covered very extensively in the literature of numerical analysis. An excellent presentation is that by Scarborough.[6] An important property of the usual power series interpolating polynomial is that the n^{th} differences of a polynomial of the n^{th} order are constant when the values of the independent variables are taken in arithmetic progression, that is, at equal intervals. This property makes it possible to represent any function, including one in tabular form, by a polynomial if its differences at some degree become constant or nearly constant.

When the tabulation is made at equal intervals, the accuracy of the quadrature formula depends upon the size of the interval, the limits of integration, and the degree of the interpolation formula.

General Quadrature Formula

Scarborough and others present a derivation of Newton's formula for forward interpolation which is the basis for developing a general quadrature formula for equidistant ordinates.[7,8,9]

This general quadrature formula is:

$$\int_{x_0}^{x_0+nh} y\,dx = h\Big[ny_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 y_0}{2!}$$

$$+ \left(\frac{n^4}{4} - n^3 + n^2\right)\frac{\Delta^3 y_0}{3!} + \left(\frac{n^5}{5} - \frac{3n^4}{2} + \frac{11n^3}{3} - 3n^2\right)\frac{\Delta^4 y_0}{4!}$$

$$+ \left(\frac{n^6}{6} - 2n^5 + \frac{35n^4}{4} - \frac{50n^3}{3} + 12n^2\right)\frac{\Delta^5 y_0}{5!}$$

$$+ \left(\frac{n^7}{7} - \frac{15n^6}{6} + 17n^5 - \frac{225n^4}{4} + \frac{274n^3}{3} - 60n^2\right)\frac{\Delta^6 y_0}{6!}\Big] \tag{2.4}$$

where n is the number of equidistant intervals of width h, and Δ is the difference operator on the value of the variable y.

The notation used for the difference operator in the general quadrature formula can be explained as follows. If

$y_0, y_1, y_2 \ldots y_n$ denote a set of values of any function $y = f(x)$, then $y_1 - y_o, y_2 - y_1, y_3 - y_2, \ldots y_n - y_{n1}$, are called the <u>first differences</u> of the variable y. Denoting these differences by Δy_0, Δy_1, Δy_2, etc., we have $\Delta y_0 = y_1 - y_0$, $\Delta y_1 = y_2 - y_1$, $\Delta y_{n-1} = y_n - y_{n-1}$, $\Delta y_n = y_{n+1} - y_n$.

The differences of the first differences are called the <u>second differences</u> and are denoted by $\Delta^2 y_0$, $\Delta^2 y_1$, etc.

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0 = y_2 - 2y_1 - y_0$$

$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1 = y_3 - 2y_2 - y_1$$

In the like manner, the third differences are:

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0 = y_3 - 3y_2 - 3y_1 - y_0$$

$$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1 = y_4 - 3y_3 - 3y_2 - y_1$$

Numerical analysis literature is replete with diagonal and horizontal difference tables. The use of these tables depends on which method of interpolation is to be used. In this study, forward interpolation will be used since the tabulated data values begin and end with a zero value over the range of integration.

From the general quadrature formula, specific quadrature formulas may be obtained by varying the value of n.

Three quadrature formulas have found widespread use due to their simplicity and accuracy. These formulas are;

Trapezoidal Rule, Simpson's Rule and Weddle's Rule. They are obtained from equation (2.4) by letting n = 1, 2, and 6 respectively.

Trapezoidal Rule

Substituting n = 1 in equation (2.4) and neglecting all differences above the first, one obtains

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_{2\ldots} + y_{n-1}) + y_n \right] \tag{2.5}$$

The geometric significance of this formula is that a given function y(x) is replaced by n/1 arcs (straight-line segments), i.e., first degree polynomials. It is exact if y = f(x) is a polynomial of degree 1.

Simpson's Rule

Substituting n = 2 in equation (2.4) and neglecting all differences above the second, one obtains

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_{3\ldots} + y_{n-1}) + 2(y_2 + y_{4\ldots} + y_{n-2}) + y_n \right] \tag{2.6}$$

Simpson's Rule corresponds to the representation of the given function y(x) by n/2 arcs of a second degree polynomial. It is exact if y = f(x) is a polynomial of degree 2 or less. While it is not readily apparent, Simpson's Rule is exact if y = f(x) is a polynomial of degree 3 because the third difference ($\Delta^3 y_0$) is zero and the first term dropped involves $\Delta^4 y_0$.

Weddle's Rule

Substituting n = 6 in equation (2.4) and neglecting all differences above the sixth, one obtains

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{3}{10}h(y_0+5(y_1\ldots+y_{n-6})+(y_2\ldots y_{n-5})+6(y_3\ldots+y_{n-4})$$

$$+ (y_4\ldots y_{n-3})+5(y_5\ldots+y_{n-2})+(y_6\ldots+y_{n-1}) \qquad (2.7)$$

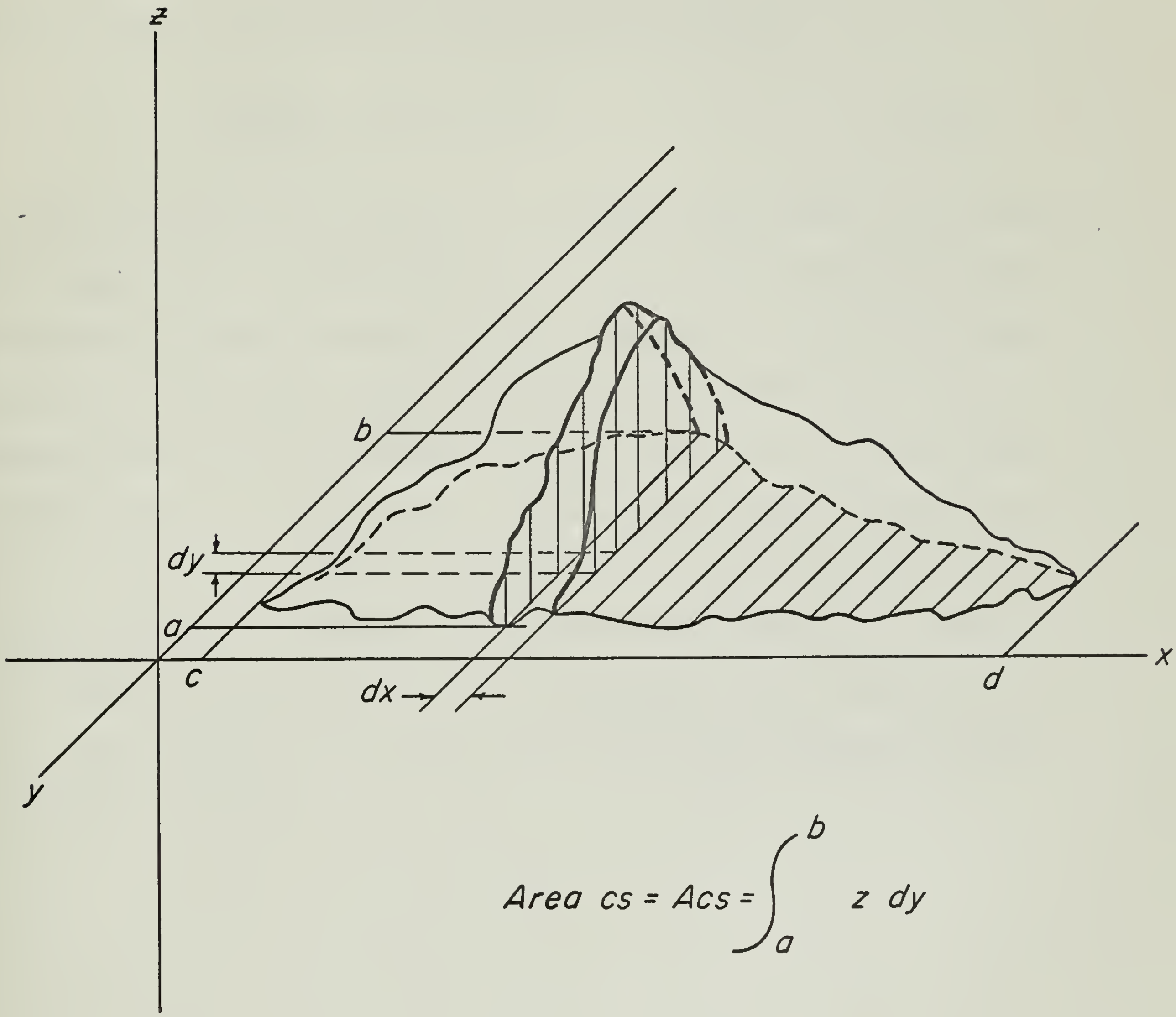
The geometric interpretation of this rule is that a given function y(x) is replaced by n/6 arcs of a fifth degree polynomial. This rule is exact for fifth-degree polynomial or lower. This formula requires at least seven consecutive values of the function, or, stated alternatively, the integration interval must consist of six subintervals of equal width h.

Volume Computations

To calculate the volume of any solid using quadrature formulas, it is only necessary to establish an x-y grid system and to determine the values of the Z coordinate corresponding to each point of that system. Figure (2.1) shows the graphical representation for this method of mechanical cubature. Thus, if a solid is divided into cross sections, each ordinate of that cross section may be measured and tabulated. These values may then be used in any acceptable quadrature formula such as Simpson's or Weddle's formulas to determine the area, A_{cs}, of each cross section.

Figure 2.1

Graphical Representation of
Mechanical Cubature on a Irregular Solid



$$\text{Area } cs = Acs = \int_a^b z\, dy$$

$$\text{Volume} = \int_c^d Acs\, dx$$

$$A_{cs} = \int_a^b Z \, dy \tag{2.8}$$

Subsequently, the cross sectional areas, A_{cs}, may be used as the ordinates of the length function and themselves be integrated using any acceptable quadrature formula.

$$\text{Volume} = \int_c^d A_{cs} \, dx \tag{2.9}$$

In this manner it is possible to compute the volume of a solid nine ways if one is limited to the three most frequently used quadrature formulas based on equal spacing of the independent variable. Generally, the accuracy of the double quadrature method depends on the number of data points and on the shape of the surface of the solid. An important characteristic of all the quadrature formulas based upon an interpolating polynomial is that the formulas are numerically exact for those instances in which an n^{th} order polynomial y(x) is used and for which instance the actual function being approximated is of order n or less.

CHAPTER III

EXPERIMENTAL PROCEDURE

Apparatus

The experimental apparatus shown in Figure (3.1) was utilized to construct and measure the different configurations of stockpiles.

The test site consisted of a 4' x 8' sheet of 3/4" plywood mounted on a large stable table. One inch ruled coordinate paper was then attached securely to the surface of the test bed. A rectangular area 24" x 60" was ruled on the coordinated paper to establish the boundary of the experimental stockpiles. These boundaries were used as the reference lines in measuring the size of the stockpile at its base.

Conveyor System

An overhead track system was devised to simulate an overhead conveyor system. A small trolley, carrying a steel funnel, was allowed controlled movement along the overhead track. The distance from the funnel spout to the test bed was designed to be eighteen (18) inches. The model conveyor duplicated the formation of stockpiles as they appear in the plant warehouse of the Cooperative Farm Chemical Association.

Measuring Device -- Depth Gauge

Two laboratory point-depth gauges, Lory Type - A, manufactured by Leupolo and Stevens Instruments, Portland, Oregon

Figure 3.1

Experimental Apparatus for
Constructing Laboratory Scale Stockpiles.

and capable of measuring distances to an accuracy of .001 foot, were mounted in two "T" frames which were constructed from two inch angle iron. This mechanism permitted the measurement of the stockpile surface at one inch intervals both across and along the length of the stockpile. The gauges were leveled and calibrated for each series of cross sectional measurements.

Materials

The ammonium nitrate used was taken from the standard 80 lb. bags supplied by the Cooperative Farm Chemical Association. A standard 80 lb. bag of ammonium nitrate was determined by repeated measurements to contain an average of 2929.0 cubic inches. The initial density was determined to be 47.19 lbs./cu.ft. Frequent checks were made to determine if the stockpile compacted during the four weeks duration of the experiment. These measurements indicated that possibly a very slight compaction occurred mostly near the center regions of the stockpile. However the change in height of the pile at all positions was within the experimental error inherent in the height measurements. Therefore, it was concluded that the density change was not sufficient to warrant its inclusion in the volume calculations. The original volume was therefore used as the basis for all calculations during the experiment.

Miscellaneous Equipment

A stone jar was used to measure the volume of the ammonium nitrate used in the investigation. The volume of

the jar was determined by measuring the volume of the water necessary to fill it. A calibrated 1000 cc cylinder was used to fill the jar. The capacity of the stone jar was determined to be 842.0 cu.in.

Measurement Procedure

A preliminary series of measurements was conducted using a single small conical pile of ammonium nitrate granules, to test the proposed measuring procedure. Initially, it was proposed that the tip of the depth gauge would contact a granule on the pile surface and thereby determine the height of the pile at that point. Measurements on the small pile using this procedure proved difficult and gave inaccurate results. This inaccuracy was caused by the irregular arrangement of the ammonium nitrate pellets or granules. The tip would displace a granule and leave the tip in the air above the pile. After repeated measuring attempts, a technique was developed whereby the sharp needle-like point of the measurement probe penetrated the stockpile surface 1/16 of an inch (approximately one pellet diameter). This method resulted in consistent height measurements.

The experiment began with the filling of the funnel with the ammonium nitrate granules. The nitrate was then allowed to drop continuously from the funnel conveyor in the overhead track system, falling the eighteen inches to the test bed. When sufficient granules had been deposited, such that the base of each sub-stockpile reached the ruled

boundaries, the funnel conveyor was then moved twelve inches to a new position. This procedure was repeated until the 80 lbs. (2929 cubic inches) of ammonium nitrate was deposited on the test site.

Figure (3.2) shows the configuration of the "undisturbed" stockpile. The four concial shaped sub-piles were formed to approximate the typical storage situation in the warehouse. The conveyor system was then removed from the test site and the height measuring mechanism installed at one end. After calibrating the height gauge, measurements were taken over the surface of the stockpile at one inch intervals in compliance with the requirements of the quadrature formulas. In addition to the 1525 height measurements, 61 width measurements were taken of the outline which the base of the stockpile made on the ruled paper. The angle of repose was also measured at various cross sections around the stockpile using a jointed protractor. Table (3.1) tabulates these measurements.

Upon completion of the measurements on the "undisturbed" stockpile, a second stockpile was created by selectively removing some material from the above mentioned "undisturbed" pile. It was felt that this would represent the configuration of a "working" or "disturbed" stockpile. A child's toy shovel was used to remove small amounts of nitrate from several positions at the base of the undisturbed pile. This process simulated quite well the random removal of the ammonium nitrate by a tractor loader. The amount of 842 cubic inches of

[illegible]

Figure 3.2

Undisturbed Laboratory Scale Stockpile.

TABLE 3.1

ANGLE OF REPOSE AT PEAK

CROSS SECTIONS AS MEASURED BY PROTRACTOR

Cross Section	Undisturbed Stockpile		Disturbed Stockpile	
	Left Side	Right Side	Left Side	Right Side
13	31.0	31.0	27.0	28.5
25	31.0	31.5	28.5	29.0
37	31.5	30.5	26.5	30.0
49	30.5	31.0	28.5	29.0
END	Left End	Right End	Left End	Right End
	31.0	30.5	27.0	27.5
Average Angle of Repose	30.9		28.2	

ammonium nitrate was removed from the base around the entire periphery of the original stockpile. Figure (3.3) shows the "disturbed" stockpile. After the removal of the 842 cubic inches of material no attempt was made to alter the shape of the pile. As can be seen from Figure (3.4), the surface tends to be linear and the peaks have regressed into ridges. The "disturbed" stockpile was then measured in the same manner as was the "undisturbed" stockpile.

Computational Models

An explanation of the various computational models employed to ascertain the volume of the "undisturbed" and of the "disturbed" stockpiles will now be given. These models range in complexity from the simplist in which the stockpile cross sections are assumed to be isosceles triangles to the most complex model wherein the depth measurements on a one inch square grid are numerically integrated using Weddle's Rule. The computer programs for each model were devised to allow parameters such as angle of repose, and the number of measurements in the x and y directions of the x-y base grid to be considered as variables, to be read in as data. This flexibility permitted a study of the influence of these parameters on the volume calculation.

The computer programs for the computational models used in this study may be found in Appendix A.

Model # 1

A computer program written in Fortran IV language was developed to compute the volume of stockpiles assuming

Figure 3.3

Disturbed Laboratory Scale Stockpile.

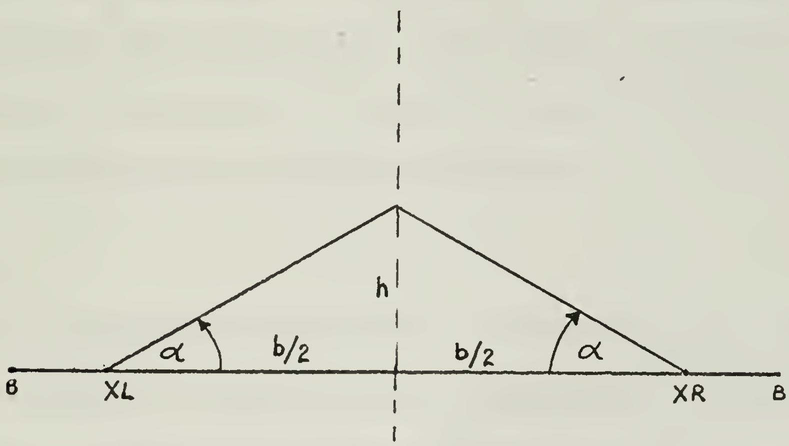
Figure 3.4

Disturbed Laboratory Scale Stockpile

Depicting the Change in the Angle of Repose.

isosceles triangular cross sections. The program calculates the cross sectional areas utilizing the angle of repose, α, as a parameter. Additionally, in computing the volume by numerically integrating the cross sectional areas over the length of the stockpile, three different quadrature formulas were used. The number of cross sections was also varied to observe its effect on the volume calculations.

The equation for the area of the isosceles triangular cross section is shown below for the circumstance in which the angle of repose, α, and the base b are known



$$\tan \alpha = h/\tfrac{b}{2}$$

$$\text{Cross Section Area} = 1/2\ bh$$

$$= (1/2)(b)(\tfrac{b}{2} \tan \alpha)$$

$$= \frac{b^2}{4} \tan \alpha \qquad (3.1)$$

Model # 2

A computer program was developed to estimate the volume of a stockpile by numerical integration in two directions. The program calculates three different areas at each cross section utilizing the three quadrature rules:

1) Trapezoidal Rule,
2) Simpson's Rule,
3) Weddle's Rule.

Also the number of depth measurements used in each cross sectional area can be designated externally and used as a model parameter. The program calculates the volume by numerical integration of the cross sectional areas previously obtained. For this phase of computation the number of cross sections and particular quadrature rule to be used can be selected externally and used as model parameters.

Model # 3

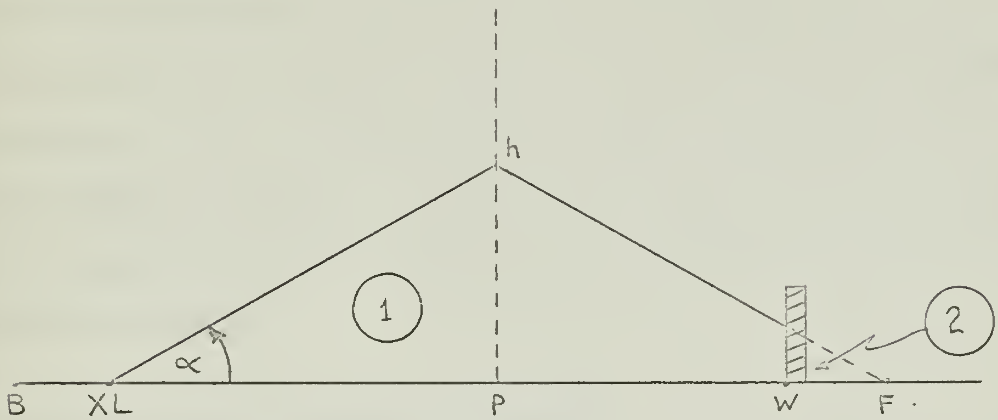
This computer program is a modification to Model #1 and was written to specifically compute the volume of a stockpile which is banked against a retaining wall.

The cross sections are assumed to be isosceles triangles but with a portion of the triangle removed. The net area of each cross section is determined by computing the area of the isosceles triangle and then subtracting that part of the isosceles triangle which projects beyond the retaining wall.

Using the angle of repose as a parameter, the program proceeds to compute the volume of the stockpile by numerical

integration of the previously obtained cross sectional areas using the three quadrature rules.

The equation for the area of a typical cross section is derived below for a banked stockpile.



$$\tan \alpha = \frac{Ph}{XLP}$$

$$\text{Area } (1) = \tfrac{1}{2}(XLP)(Ph)$$

$$= \tfrac{1}{2}(XLP)^2 \tan \alpha$$

$$\text{Area } (2) = \tfrac{1}{2}(WF)^2 \tan \alpha$$

$$\text{Net Area} = 2 \times \text{Area } 1 - \text{Area } 2$$

$$= (XLP)^2 \tan \alpha - \tfrac{1}{2}(WF)^2 \tan \alpha \tag{3.2}$$

CHAPTER IV

ANALYSIS OF EXPERIMENTAL RESULTS

General

The results of the experimental study outlined in Chapter III will be presented in three parts. Part A is concerned with the computation of the volume and the determination of the apparent angle of repose for the "undisturbed" stockpile. Part B describes the same results but for the "disturbed" stockpile. Part C describes the results from the determination of the true angle of repose for coated ammonium nitrate granules.

A. Undisturbed Stockpile Results

Computational Models #1 and #2 were used in obtaining the volume and apparent angle of repose approximations.

Model #1

Table (4.1) tabulates the per cent error in the calculated volume obtained from computational Model #1 using as parameters;

1. number of isosceles triangle cross sections
2. quadrature rule
3. apparent angle of repose.

Because of the mathematical form of Model #1, the apparent angle of repose, α, is an independent parameter. The volume of the stockpile computed by the model is dependent upon the value selected for α. If the volume of the ammonium nitrate

TABLE 4.1

PER CENT ERROR IN CALCULATED VOLUME FOR UNDISTURBED STOCKPILE USING COMPUTATION MODEL #1

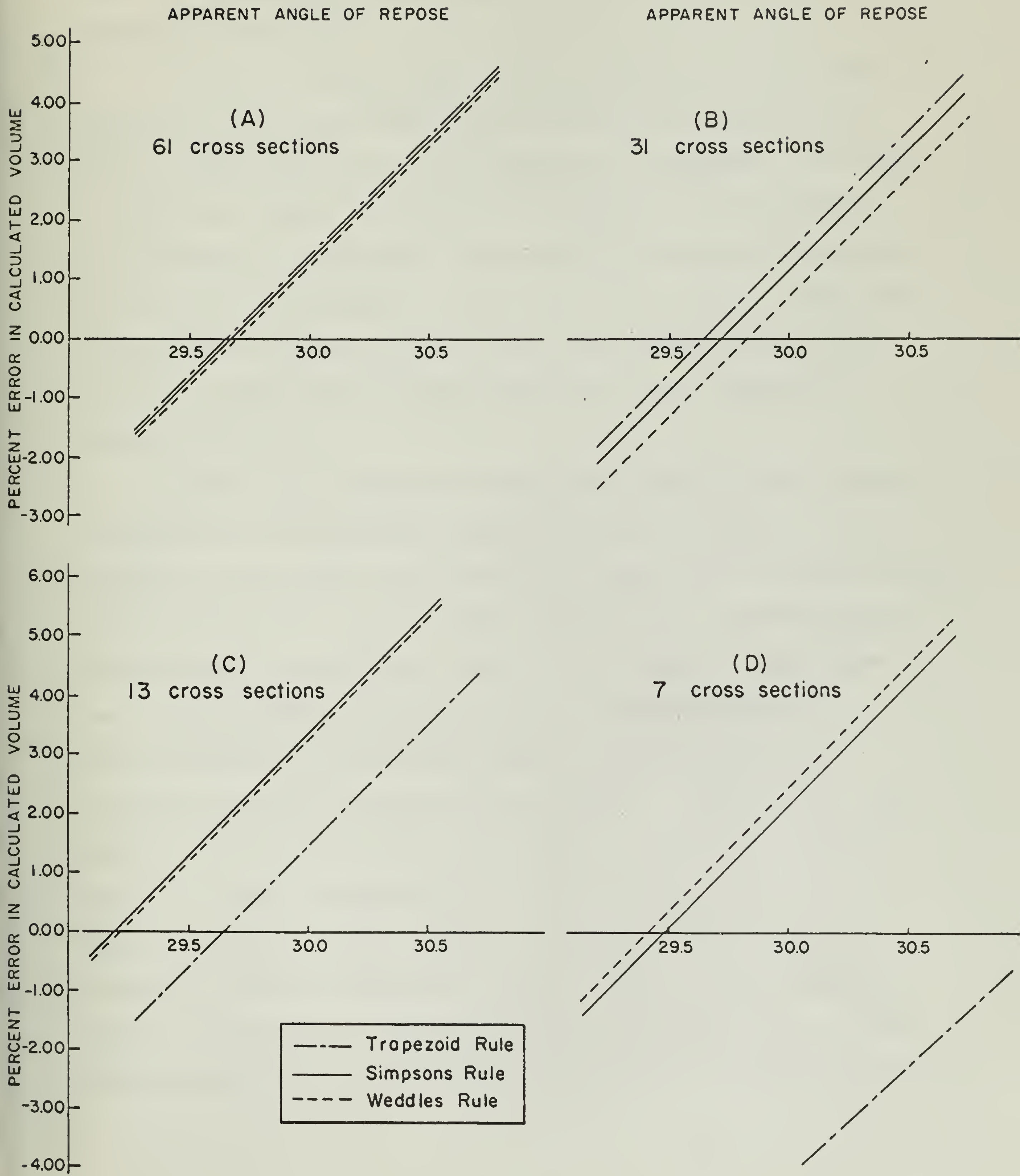
Cross Sections	Per Cent Error in Calculated Volume			
	Apparent angle of repose	Trapezoidal Rule	Simpson's Rule	Weddle's Rule
61	31.0	5.49	5.47	5.45
	30.0	1.36	1.35	1.32
	29.68	--	--	0.00
	29.67	--	0.00	--
	29.66	0.00	--	--
	29.00	-2.68	-2.70	-2.72
	28.0	-6.65	-6.67	-6.69
	27.0	-10.55	-10.56	-10.58
31	31.0	5.54	5.20	4.72
	30.0	1.41	1.08	0.62
	29.85	--	--	0.00
	29.71	--	0.00	--
	29.65	0.00	--	--
	29.0	-2.63	-2.95	-3.39
	28.0	-6.60	-6.91	-7.33
	27.0	-10.50	-10.79	-11.20
13	31.0	5.56	7.51	7.41
	30.0	1.43	3.30	3.21
	29.65	0.00	--	--
	29.22	--	--	0.00
	29.20	--	0.00	--
	29.0	-2.62	-0.82	-0.91
	28.0	-6.59	-4.87	-4.95
	27.0	-10.49	-8.83	-8.92
7	31.05	0.00	--	--
	31.0	-0.28	6.28	6.60
	30.0	-4.18	2.12	2.43
	29.48	--	0.00	--
	29.41	--	--	0.00
	29.0	-8.00	-1.95	-1.66
	28.0	-11.75	-5.95	-5.67
	27.0	-15.44	-9.87	-9.61

as measured in the volumetric jar is taken as correct, then the per cent error in volume estimated by Model #1 can be indicated as a function of α. The results are depicted in Figure (4.1).

Before analyzing these results it would seem appropriate again to describe the manner in which the measurements were taken. A rectangular grid boundary, 24 inches by 60 inches was drawn around the base of the stockpile. Thus, sixty-one cross sections could be selected, at a one inch spacing, the first and last cross sections having zero cross sectional area. By measuring the distance between the base of the stockpile and the side boundary for each cross section, the base length of each individual cross section was determined. The accuracy of these measurements was approximately 1/10 of an inch because the scale was ruled in tenths and because individual particles were not much smaller than 1/10 inch.

From Figure (4.1a), it may be observed that the three quadrature rules give essentially the same error curve for volume determinations when the maximum number (61) of sectional areas is used in the computation. From the graph, the average apparent angle of repose for coated ammonium nitrate granules was determined to be approximately 29.7 degrees. The angle was taken to be the point at which the per cent error in the volume determination is zero. This apparent angle of repose would be used in Model #1 to estimate the volume of an actual undisturbed stockpile having

Figure 4.1
Error in Volume Estimation by Model Number I
for an Undisturbed Stockpile



essentially the shape of the one used in this particular test. The stockpile could consist of one or more peaks all essentially in line.

Further study of Figure (4.1) reveals that the error curves for the three integration rules used in Model #1 diverge as the number of cross sections is decreased. In order to study the effect of varying the number of cross section measurements on the volume calculation, additional volume calculations were made for 21, 16, 11, 6, 5, 4, and 3 cross sections. These results together with the results for 61, 31, 13, and 7 cross sections from Table (4.1) are listed in Table (4.2).

Table (4.2) presents the per cent error in the calculated volume utilizing the apparent angle of repose of 29.7 degrees. The results are presented graphically in Figure (4.2). The per cent error in the volume behaves as a variable undergoing damped oscillation. The error is large in magnitude for a small number of cross sections becoming damped to a small constant error as the number of cross sections approaches one-half of the total available. Specifically, the figure indicates that the error curves converge at approximately 31 cross sections and are quite close when 15 to 31 cross sections are used. Thus, to compute the volume of a stockpile having essentially the same shape as the stockpile used in the present laboratory study one should use at least 20 cross sections and possibly as many as 31 if the volume is

TABLE 4.2

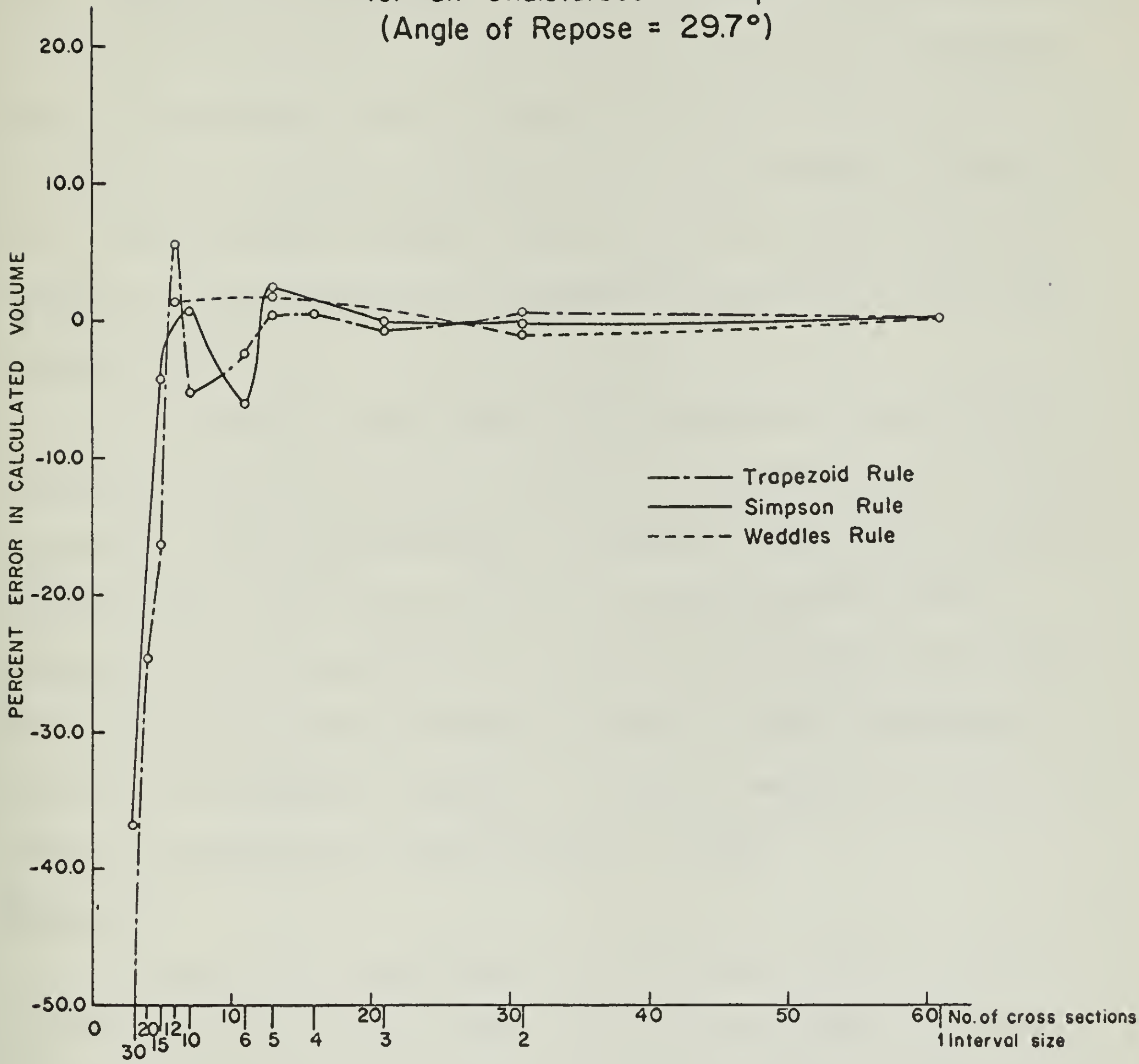
PER CENT ERROR IN CALCULATED VOLUME FOR AN APPARENT ANGLE OF REPOSE OF 29.7 DEGREES FOR MODEL #1

	Per Cent Error in Volume		
Cross Section/ Interval Size*	Trapezoidal Rule	Simpson's Rule	Weddle's Rule
61/1	0.00	0.00	0.00
31/2	0.20	-0.05	-0.52
21/3	-0.82	-0.21	--
16/4	0.31	--	--
13/5	0.20	2.10	1.97
11/6	-2.52	-6.04	--
7/10	-5.30	0.90	1.20
6/12	6.60	--	--
5/15	-16.20	-4.07	--
4/20	-24.70	--	--
3/30	-52.20	-36.80	--

* The notation 61/1, 31/2, etc., indicates 1 interval containing 61 points, 2 intervals containing 31 points each, etc. Note that the terminal point on one interval is the initial point on the next interval, thus (points -1) x no. of intervals = 60 for all sets.

Figure 4.2

Error in Volume Estimation by Model Number I
for an Undisturbed Stockpile
(Angle of Repose = 29.7°)



to be estimated by Model #1. The appropriate apparent angle of repose would, of course, be 29.70 degrees.

It is believed that the results of the above laboratory model can be scaled to the dimensions of commercial stockpiles by considering the number of peaks in the commercial undisturbed stockpile. Thus, assuming the minimum number of cross sections is 20 for the 4 peaked pile studied above, (see Figure 3.2) one might generalize the results to a rule that a minimum number of 5 cross sections per peak, i.e., 20 sections/4 peaks, should be used to estimate the volumes of undisturbed multi-peaked stockpiles of the type studied herein.

It is to be noted that the Trapezoidal Rule gives a relatively low error level over the greatest range of cross sections. The Trapezoid Rule calculates the volume with an accuracy of $\pm$ 1.0% for 13 or more cross sections while Simpson's Rule calculates the volume to this same accuracy for 21 or more cross sections. Weddle's Rule converges slower and therefore requires a greater number of cross sections to compute the volume within $\pm$ 1.0%.

<u>Model #2</u>

Table (4.3) tabulates the per cent error in the calculated volume obtained from computational Model #2 which involves two-way numerical integration. The following parameters were varied in this study:

1. number of cross sections

TABLE 4.3

PER CENT ERROR IN CALCULATED VOLUME FOR UNDISTURBED STOCKPILE USING COMPUTATION MODEL #2

		Number of Depth Measurements			
		25 PTS	13 PTS	7 PTS	3 PTS
Cross Sections	Integration Rule*	% Error Calc. Volume	% Error Calc. Volume	% Error Calc. Volume	% Error Calc. Volume
61	TRTR	0.59	0.60	1.67	2.57
	TRSN	0.65	0.67	1.74	2.65
	TRWD	0.70	0.71	1.78	2.72
	SNTR	0.59	0.24	2.35	36.76
	SNSN	0.65	0.31	2.41	36.86
	SNWD	0.69	0.35	2.46	36.96
	WDTR	0.63	0.16	2.98	99.94
	WDSN	0.69	0.23	3.05	100.08
	WDWD	0.73	0.27	3.10	100.22
31	TRTR	0.39	0.39	1.47	2.34
	TRSN	0.20	0.24	1.25	2.14
	TRWD	-0.23	0.14	0.75	1.85
	SNTR	0.39	0.02	2.15	36.45
	SNSN	0.19	0.10	1.92	36.18
	SNWD	-0.26	-0.44	1.40	35.80
	WDTR	0.43	-0.06	2.78	99.49
	WDSN	0.22	-0.15	2.54	99.02
	WDWD	-0.24	-0.45	2.03	98.36
13	TRTR	1.09	1.11	2.19	2.65
	TRSN	3.74	3.82	4.77	6.13
	TRWD	4.59	4.70	5.57	7.62
	SNTR	1.08	0.76	2.53	36.87
	SNSN	3.72	3.50	5.20	41.51
	SNWD	4.55	4.41	6.10	43.49
	WDTR	1.10	0.66	2.85	100.13
	WDSN	3.74	3.42	5.68	106.59
	WDWD	4.58	4.36	6.74	109.24
7	TRTR	-6.88	-6.99	-5.57	-7.77
	TRSN	0.64	0.54	2.02	-0.33
	TRWD	1.60	1.52	2.94	0.88
	SNTR	-6.85	-7.46	-5.48	22.98
	SNSN	0.68	0.05	2.16	32.90
	SNWD	1.63	1.04	3.09	34.51
	WDTR	-6.82	-7.62	-5.63	80.72
	WDSN	0.70	-0.10	2.04	95.17
	WDWD	1.63	0.92	3.01	97.36

*TR = Trapezoid Rule, SN = Simpson's Rule WD = Weddle's Rule, TRSN = Cross section integration using Trapezoid Rule and length integration using Simpson's Rule

2. number of depth measurements per cross section
3. quadrature rule for cross sections
4. quadrature rule for length

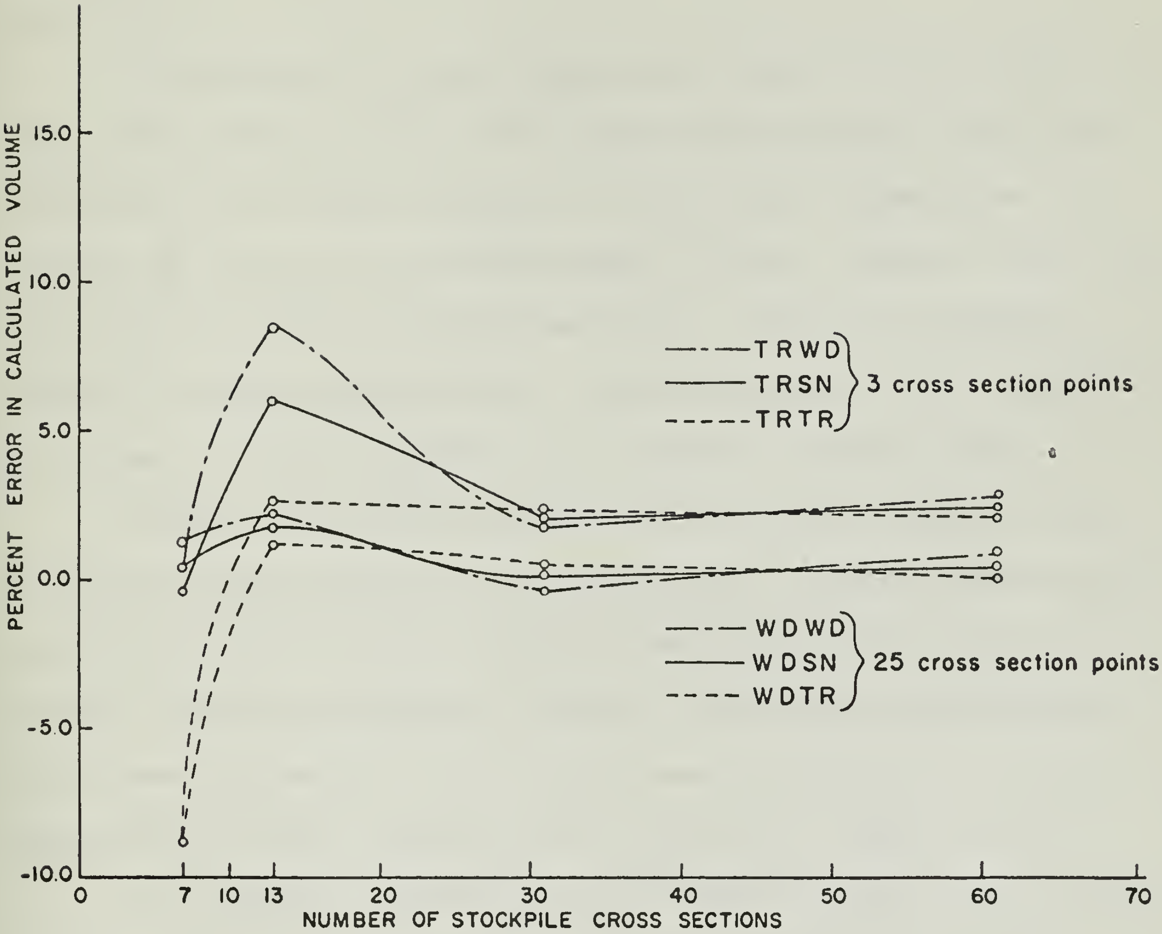
Because of the difficulty of conveniently representing graphically all of the data contained in Table (4.3) only certain extreme examples are plotted herein. These correspond to the three curves which give consistently the smallest error in the volume calculation when using both the maximum number of points (25 points) in the cross sectional area determination and the minimum number of cross section points (3 points). These results are plotted as Figure (4.3).

It may be seen that the error curves have the same damped oscillation feature observed in Model #1 with the error becoming relatively constant when approximately one half of the total possible cross sections on a one inch spacing are used, regardless of the number of depth measurements per cross section. As might be expected, the error is greater for the smaller number of depth measurements used per cross section.

The results of Figure (4.3) show that a remarkably low error is incurred by using as few as three points per cross section when a large number of lengthwise cross sections are taken. This error, less than 3%, is not appreciably greater than that obtained with a large number of cross sections and the assumption of an isosceles triangular cross section using Model #1.

Figure 4.3

Error in Volume Estimation by Model Number 2 for an Undisturbed Stockpile



This observation seems to indicate that, for Model #2, one need not use a large number of data points in both the width and length dimensions on an undisturbed stockpile to obtain a satisfactory estimate of stockpile volume. Instead, one is probably justified in using a rather elementary approach in obtaining cross sectional areas provided a sufficient number of these cross sections is used for the lengthwise integration.

An analysis of the calculated areas for two typical sections was made to determine the best method of obtaining the area of any cross section. Table (4.4) tabulates the results of the area determinations for two typical cross sections for the three quadrature rules. Using the maximum number of depth measurements, all quadrature rules computed the areas for the typical cross sections within .50%.

Using the results obtained by applying Weddle's Rule to the maximum number of cross sectional depths measurements as the "correct" answer and then comparing the results obtained from both Simpson's and the Trapezoidal quadrature Rules based upon only three depth measurmments, it may be observed that the Trapezoid Rule gives the lowest error.

In a study similar to that shown in Table (4.4), but involving more cross sections, it was revealed that the errors in area estimation by the Trapezoidal Rule were opposite in sign for cross sections of high and of low area. These errors tended to cancel when used in the lengthwise integration formulas, thus further substantiating the use

TABLE 4.4

COMPARISON OF COMPUTED CROSS SECTIONAL AREAS FOR UNDISTURBED STOCKPILE

Parameters	Cross Sectional Area (in^2)		
Number of Depth Measurements per Cross Section	Quadrature Rule	Number 13 (large)	Number 7 (small)
25	Weddle's	74.66	27.97
25	Simpson's	74.62	27.90
25	Trapezoidal	74.62	27.86
3	Trapezoidal	72.00	31.39
3	Simpson's	96.00	41.86

Considering the 25-point Weddle's Rule values to be the "correct" areas for each cross section the error at selected cross sections are as follows:

Cross Section #13

$$\text{Per Cent Error in Area} = \frac{(72.00 - 74.66)}{74.66} \, 100.0 = -3.56\%$$

or Error in square inches = -2.66.

Cross Section #7

$$\text{Per Cent Error in Area} = \frac{(31.39 - 27.97)}{27.97} \, 100.0 = +12.2\%$$

or Error in square inches = +3.42.

of the simple Trapezoidal Rule for obtaining cross sectional areas.

Table (4.5) summarizes for Model #2 the best alternatives in keeping with the basic purpose of this study - namely to determine the minimum number of measurement points and their location in order to best approximate the volume of a stockpile within the prescribed error range. The table presents the alternatives for which the volume determination on the undisturbed stockpile, computed by Model #2, is less than $\pm$ 3%.

From the observations of the volume calculation methods for the undisturbed stockpile it may be stated that both Models #1 and #2 give excellent results when utilizing a maximum number of data measurements. Also, under appropriate circumstances, a much smaller number of measurements can be used in the models and still maintain the error levels within a tolerable range. Specifically, one is justified in using a rather simple geometrical representation in the cross-wise or cross sectional area determinations for elongated undisturbed stockpiles. It is possible to determine the cross sectional areas with a high degree of accuracy from as few as 3 to 5 measurements, with approximately 5 cross sections being taken per peak over the length of the stockpile.

If this procedure is followed using the Trapezoid or Simpson's quadrature Rule, then the stockpile volume should not be in error by more than $\pm$ 3%. Further refinements are, of course, possible using additional measurements but

TABLE 4.5

ALTERNATIVE MEASUREMENTS ON AN UNDISTURBED STOCKPILE FOR VOLUME ERROR RANGE OF $\pm$ 3.0%

Number of Depth Measurements per Cross Section	Number of Cross Sections	Total Number of Measurements	Double Quadrature Rule*
25	61	1525	WDWD WDSN WDTR
25	51	775	WDSN WDTR WDWD
25	7	175	WDSN WDWD
3(1)	13	39(11)	TRTR
3(1)	7	21(5)	TRSN TRWD

Note:

Figures in parenthesis are actual number of measurements required since the end or outer boundary measurements are always zero.

*The Code for the Double Quadrature Rule is as follows:

WD = Weddle's Rule

SN = Simpson's Rule

TR = Trapezoidal Rule

thus WDSN means Weddle's Rule used in integrating the cross section and Simpson's Rule used to integrate over the length.

accuracies greater than 1% are not to be expected. On the other hand, a very rapid approximation method using only 3 points (in reality only 1 point) per cross section and 3 cross sections per peak will give an error of less than 10%.

B. Disturbed Stockpile Results

Computational Models #1 and #2 were again used in obtaining the volume and apparent angle of repose of the disturbed stockpile. The method of analysis was essentially the same as that for Part A of this chapter.

Model #1

Table (4.6) presents the per cent error in the calculated volume obtained from computational Model #1 using the following as parameters;

1. number of isosceles triangle cross sections
2. quadrature rule
3. apparent angle of repose.

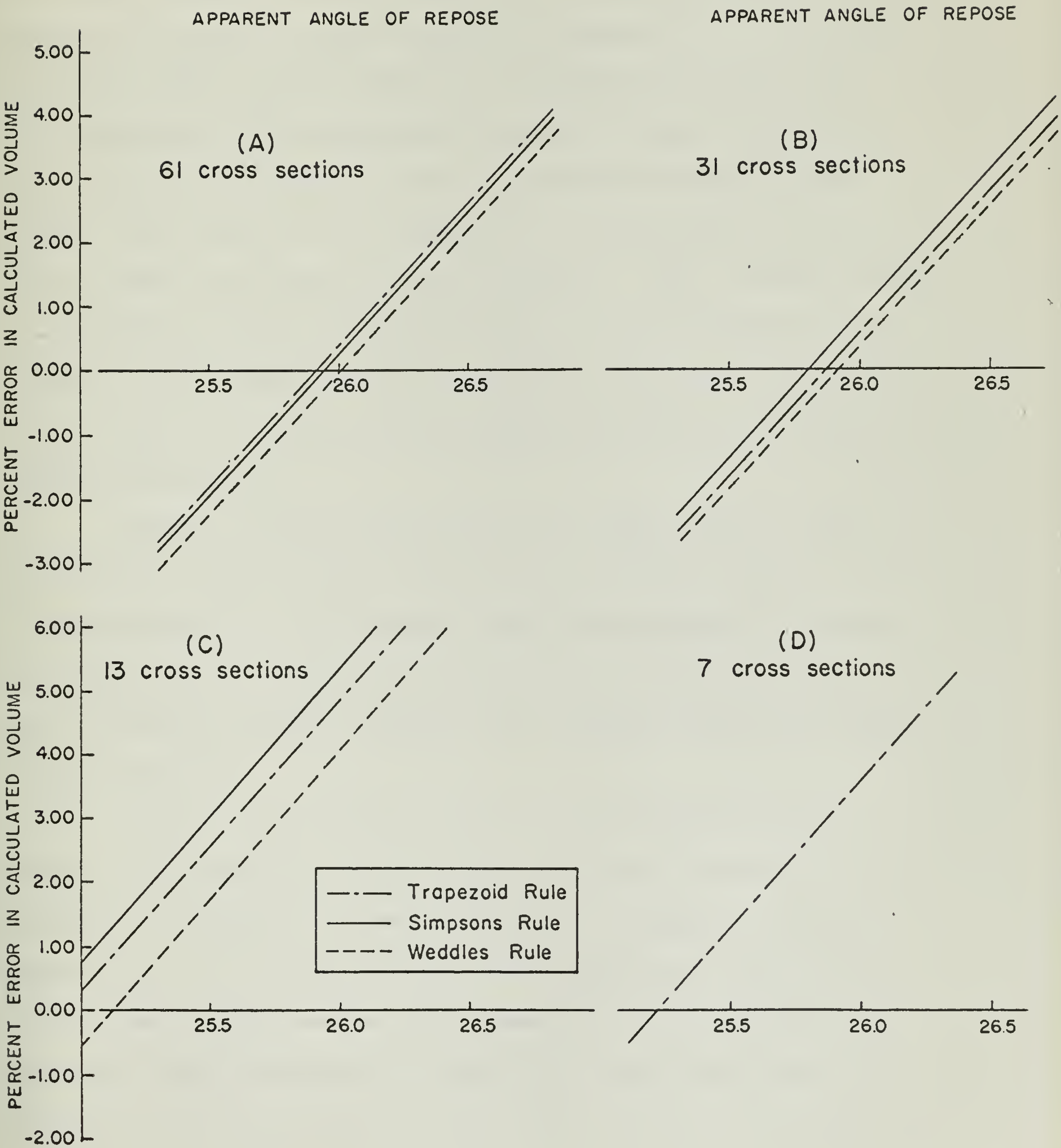
These results are shown graphically by Figure (4.4). It can again be observed that the error in the volume estimation by the three quadrature rules is essentially the same when the maximum number of cross sections is used in the computation. The graphs show that the apparent angle of repose for a disturbed stockpile is approximately 25.9 degrees. This value closely corresponds to the value of 26.5 degrees used by the Cooperative Farm Chemical Association in their calculations on disturbed stockpiles. Further observations of Figure (4.4) shows that the error curves are less widely separated for the

TABLE 4.6

PER CENT ERROR IN CALCULATED VOLUME FOR DISTURBED STOCKPILE USING COMPUTATION MODEL #1

Cross Sections	Per Cent Error in Calculated Volume			
	Apparent Angle of Repose	Trapezoidal Rule	Simpson's Rule	Weddle's Rule
61	27.00	4.80	4.74	4.41
	26.01	--	--	0.00
	26.0	0.32	0.26	-0.06
	25.94	--	0.00	--
	25.93	0.00	--	--
	25.00	-4.09	-4.14	-4.45
31	27.00	4.99	5.31	4.81
	26.00	0.50	0.81	0.32
	25.93	--	--	0.00
	25.89	0.00	--	--
	25.82	--	0.00	--
	25.00	-3.92	-3.62	-4.08
13	27.00	9.65	10.07	8.64
	26.00	4.96	5.36	4.00
	25.13	--	--	0.00
	25.00	0.35	0.73	-0.57
	24.92	0.00	--	--
	24.84	--	0.00	--
7	27.00	8.39	16.29	17.39
	26.00	3.75	11.32	12.37
	25.18	0.00	--	--
	25.00	-0.81	6.43	7.43

Figure 4.4
Error in Volume Estimation by Model Number I
for a Disturbed Stockpile



range of 7 to 61 cross sections than the corresponding curves applicable to the undisturbed stockpile. This implies a more nearly constant apparent angle of repose as the number of cross sections is decreased from the maximum of 61. This may be attributed to the increased homogeneity of the cross sections caused by the removal of material from the base. By reference to the disturbed stockpile, Figure (3.3) one can see that the peaks of the undisturbed stockpile have formed into ridges as a consequence of the removal of material from around the bottom of the stockpile. This action created a relatively planar stockpile surface in the region of the ridges. The forming of the ridges from each peak is therefore, visual evidence of the homogeneous structure of disturbed stockpiles.

Further volume calculations were made for the same additional cross sections as in Part A and the results are listed in Table (4.7) together with the applicable results of Table (4.6). Table (4.7) presents the per cent error in the calculated volume utilizing the apparent angle of repose of 25.9 degrees.

Figure (4.5) depicts the error in the calculated volume as a function of the number of cross sections. For both the disturbed and the undisturbed stockpiles, approximately 20 to 30 cross sections seem sufficient for an accurate volume determination with possibly slightly more cross sections being required for disturbed stockpiles since the curve for the disturbed stockpile does not seem to damp quite

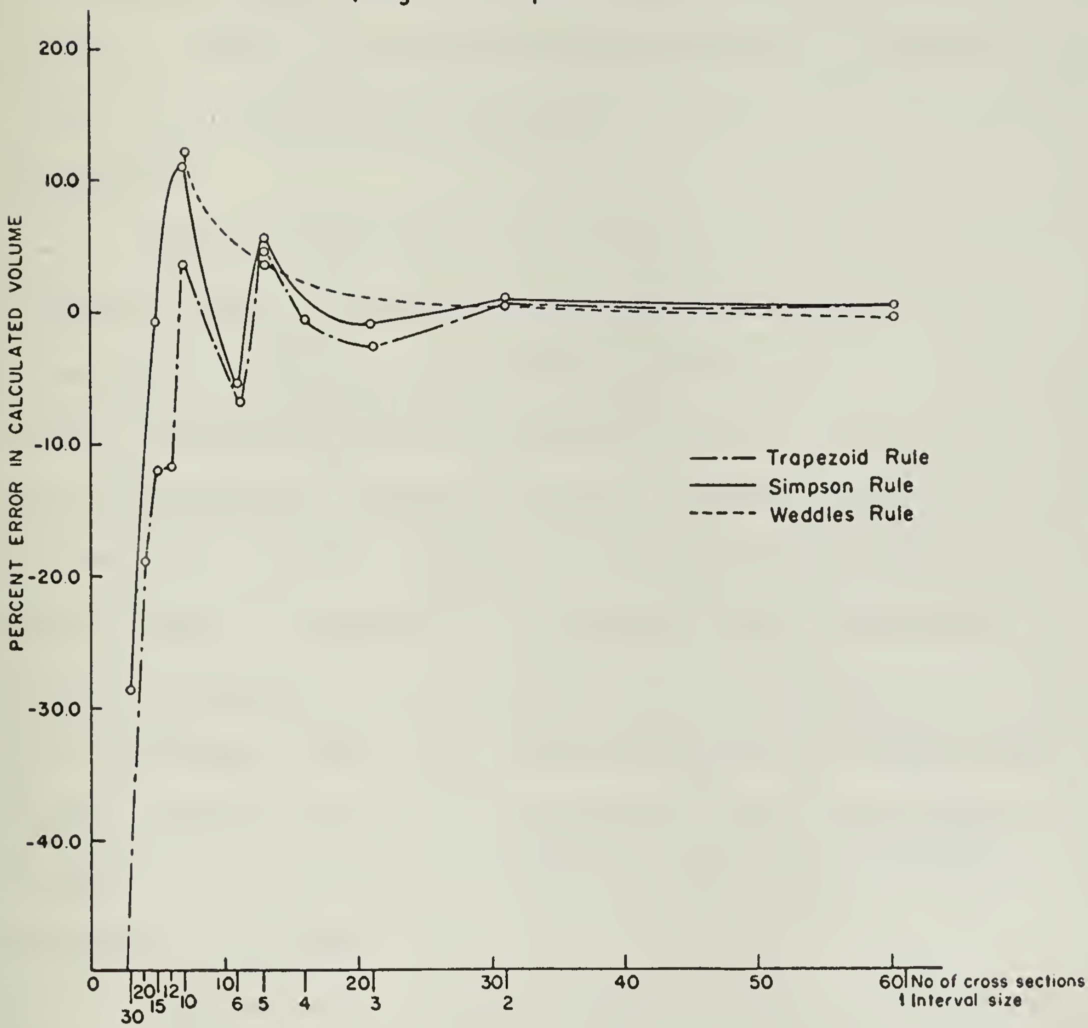
TABLE 4.7

PER CENT ERROR IN CALCULATED VOLUME FOR
APPARENT ANGLE OF REPOSE OF 25.9 DEGREES

Cross Section/ Interval	Trapezoidal Rule	Simpson's Rule	Weddle's Rule
61/1	.20	.00	-0.20
31/2	.30	.60	0.10
21/3	-2.44	-0.95	--
16/4	-0.62	--	--
13/5	4.95	5.35	4.0
11/6	-6.80	-5.20	--
7/10	3.75	11.35	12.3
6/12	-11.40	--	--
5/15	-11.80	-0.52	--
4/20	-18.90	--	--
3/30	-69.00	-28.4	--

Figure 4.5

Error in Volume Estimation by Model Number I
for a Disturbed Stockpile
(Angle of Repose = 25.9°)



as rapidly as for the undisturbed stockpile. It is not known however, how general is this difference in damping between the two types of stockpiles. For Model #1, an accuracy of ± 3% is attainable with as few as 20 cross sectional measurements.

In general, it may be concluded that the disturbed stockpiles require a slightly larger number of cross section intervals than do undisturbed stockpiles to compute the volume with the same accuracy.

Model #2

Table (4.8) presents the per cent error in the calculated volume obtained from computation Model #2 using the same parameters for Model #2 as used in Part A. A comparison of these results with those listed in Table (4.3) for the undisturbed stockpile shows a general overall increase in the error of the computed volume for the disturbed stockpile. This is especially noticeable when the number of depth measurements utilized in the computation, is decreased to a minimum. The error involving Weddle's Rule using three depth measurements is an exception; the error value is meaningless with this small number of points since Weddle's Rule requires 7 points.

Figure (4.6) depicts the per cent error in the calculated volume obtained from the double quadrature rules which gave the minimum error calculation. Two sets of error curves are plotted; one utilizing the maximum number of depth measurements per cross section (23 points) and the

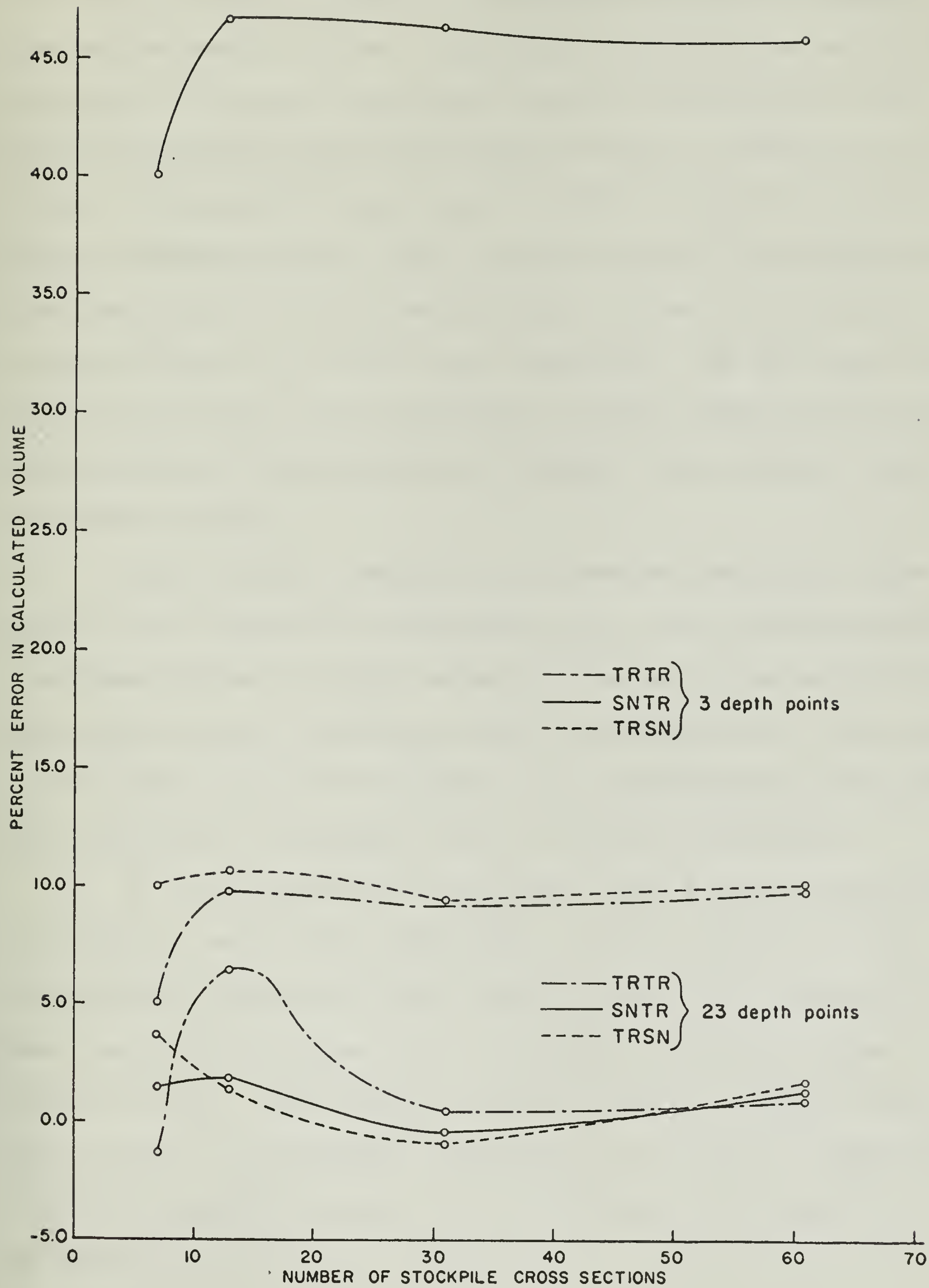
TABLE 4.8

PER CENT ERROR IN CALCULATED VOLUME FOR DISTURBED STOCKPILE USING COMPUTATION MODEL #2

		Depth Measurements			
		23 PTS	13 PTS	7 PTS	3 PTS
Cross Sections	Integration Rule	% Error Calc. Volume	% Error Calc. Volume	% Error Calc. Volume	% Error Calc. Volume
61	TRTR	0.97	1.25	2.33	9.53
	TRSN	1.51	1.80	2.96	9.97
	TRWD	1.96	2.26	3.53	10.37
	SNTR	1.06	0.89	3.38	46.05
	SNSN	1.61	1.41	4.04	46.63
	SNWD	2.06	1.84	4.63	47.16
	WDTR	1.09	1.26	6.86	64.30
	WDSN	1.64	1.76	7.53	64.96
	WDWD	2.11	2.17	8.13	65.56
31	TRTR	0.31	-0.39	0.42	9.65
	TRSN	-1.10	-0.79	0.06	9.20
	TRWD	-1.77	-1.47	-0.61	8.68
	SNTR	-0.55	-0.66	1.39	46.83
	SNSN	-0.15	-1.07	1.01	46.61
	SNWD	-1.88	-1.75	0.23	46.22
	WDTR	-0.72	-0.22	4.85	64.75
	WDSN	-1.14	-0.66	4.40	64.22
	WDWD	-1.88	-0.34	3.50	63.36
13	TRTR	6.44	1.87	3.01	9.65
	TRSN	1.34	3.01	4.50	11.10
	TRWD	2.62	3.09	4.92	12.52
	SNTR	1.72	1.49	4.28	46.90
	SNSN	2.53	2.52	6.02	49.66
	SNWD	2.46	2.48	6.67	51.14
	WDTR	1.54	1.83	7.92	64.99
	WDSN	2.51	2.83	10.03	68.51
	WDWD	2.41	2.76	10.94	73.62
7	TRTR	-1.44	-1.56	-1.46	5.00
	TRSN	3.65	3.56	3.73	10.06
	TRWD	3.22	3.14	3.31	9.11
	SNTR	-1.40	-1.59	-0.96	40.00
	SNSN	3.69	3.50	4.26	46.75
	SNWD	3.25	3.09	3.68	45.48
	WDTR	-1.35	-1.17	1.59	57.50
	WDSN	3.74	3.93	7.00	65.09
	WDWD	3.30	3.54	6.30	63.37

Figure 4.6

Error in Volume Estimation by Model Number 2
for a disturbed Stockpile



other using the minimum number of depth measurements per cross section (3 points). The dampning characteristic for both sets of error curves is somewhat diminished compared to the corresponding curves (see Figure 4.3) for the undisturbed stockpile. The most noticeable trait of this plot is the relatively large difference between these two sets of error curves. The set of error curves using the maximum number of depth measurements per cross section shows a ± 2% error for the range of 31 to 61 cross sections and an overall error level of ± 6% for 7 to 61 cross sections. On the other hand, the set of error curves utilizing 3 depth measurements give a minimum error of approximately 10% over the complete range of cross sections.

These results indicate that two-way numerical integration over disturbed stockpile is not very accurate using a minimum number of depth measurements per cross section unless one uses the Trapezoid Rule to determine the cross sectional areas and then integrate the resultant areas by either the Trapezoid or Simpson's Rule to obtain the volume.

It can therefore be concluded that for the disturbed stockpiles, which have relatively straight surfaces, the Trapezoidal quadrature Rule should be used for integration when using the minimum number of depth measurements per cross section. The volume error level which results using this method is reasonably stable but is of such a magnitude (10%) that another method for calculating the volume is to be preferred.

Table (4.9) presents for Model #2 for a disturbed stockpile, the best alternative measurements required to calculate the error in the volume to an accuracy of + 3.0%. A comparison of this table with the corresponding Table (4.5) in Part A shows that a greater number of depth measurements would be required on a disturbed stockpile to achieve the same 3.% error level.

This study has shown that numerical integration when using a minimum number of cross sectional depth measurements resulted in a error level that was 3 times greater than that determined by Model #1 using isosceles triangular cross sections. It is therefore concluded that the use of the single isosceles triangle cross section is justified in approximating the volume of disturbed stockpiles.

In summary it may be stated that Model #1 seems preferable for computing the volume of disturbed stockpiles.

C. Angle of Repose Results

The angle of repose of a granular stockpile is defined as the angle which a plane, tangent to the surface of the stockpile, makes with a horizontal plane. In this experiment, the undisturbed stockpiles consisted of sections that were conical and for this geometrical shape the angle of repose corresponds to the angle formed by the horizontal plane and by a line lying in the conical surface and passing through the apex of the cone.

Since height measurements were taken at one inch intervals, in general, a cross section would not pass precisely

TABLE 4.9

ALTERNATIVE MEASUREMENTS ON A DISTURBED STOCKPILE
FOR A VOLUME ERROR RANGE OF ± 3.0%

Number of Depth Measurements per Cross Section	Number of Cross Sections	Total Number of Measurements	Double Quadrature Rule*
23	61	1403	TRTR SNTR TRSN
23	31	713	SNSN TRTR TRSN
23	7	161	TRSN SNTR SNWD
7(5)	31	217(155)	TRSN TRTR TRWD
7(5)	13	91(65)	TRTR
7(5)	7	49(35)	SNTR TRTR

Note:

Figures in parenthesis are actual number of measurements required.

through the apex of any of the four conically shaped peaks of the undisturbed stockpile. Therefore, in order to estimate the true angle of repose it was necessary to interpolate between the cross sections bracketing the cross section which passed through the apex.

A straight line was passed through the depth measurements, by the method of least squares, for each side of all cross sections. The slope of each line was determined and the angle less than 90 degrees corresponding to this slope was computed. These angles were then plotted versus the cross section position. The true angle of repose was then taken to be the interpolated angle corresponding to the known peaks cross section position. Figure (4.7) shows a typical plot of angles computed in the manner described above and corresponding to cross sections near as undisturbed stockpile peak. The graph shows that the angle of repose is about 31.6 degrees at cross section location 13.2. Table (3.1) lists the observed angle of repose by protractor measurement for cross section number 13 as 31.0 degrees. Figure (4.8) shows a plot of the height measurements for cross section 13 together with the straight line obtained by the method of least squares.

The angle of repose was also determined in the same manner, for the disturbed stockpile. Figure (4.9) is the plot of the angles of repose for the cross sections forming the first ridge section (observed at the left in Figure 3.3) Since the apex of the conical shape degenerated to a ridge,

Figure 4.7

Angle of Repose for Undisturbed Stockpile

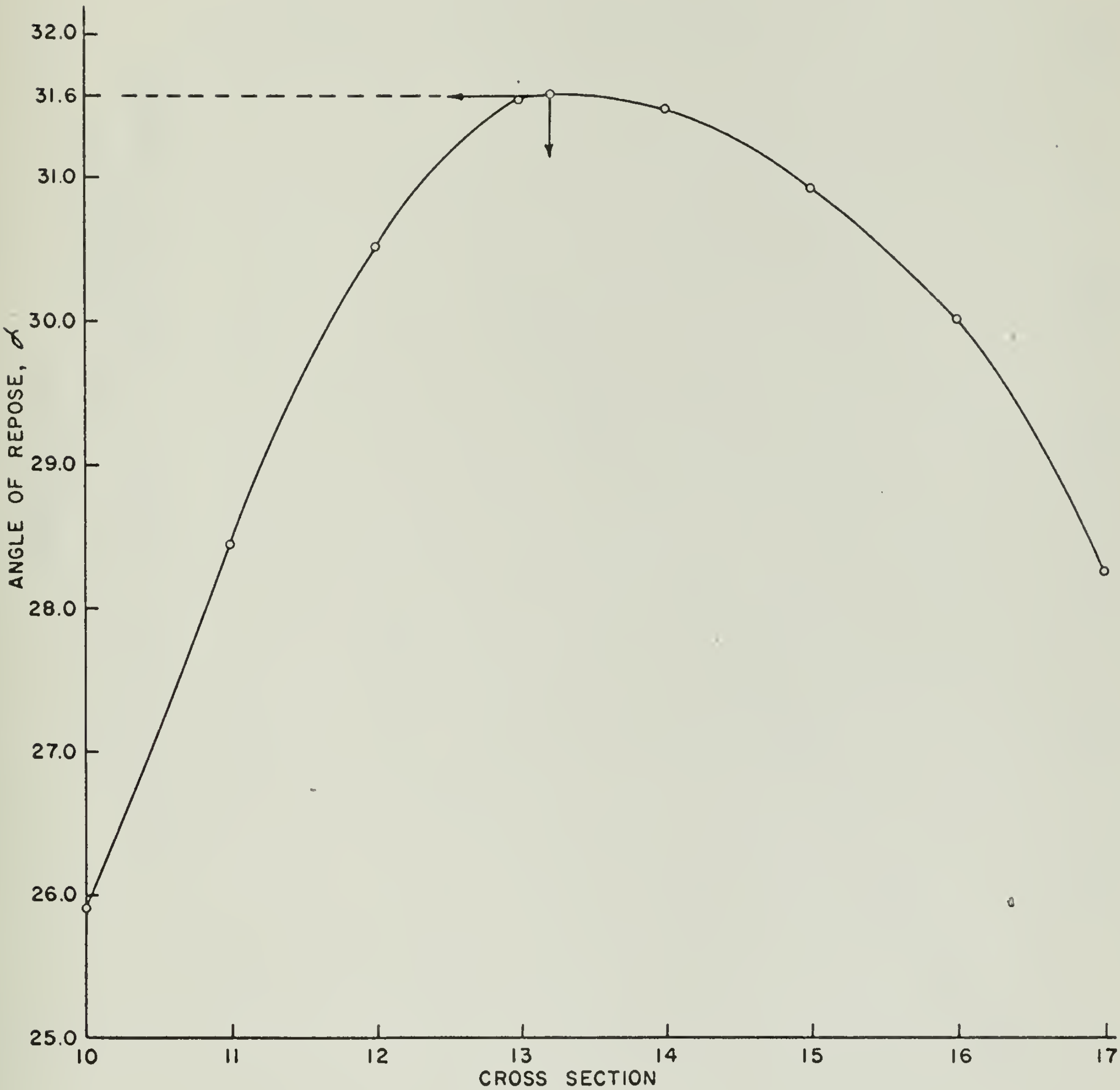


Figure 4.8

Least Squares Fit of Depth Measurements
for Cross Section Number 13 Undisturbed Stockpile

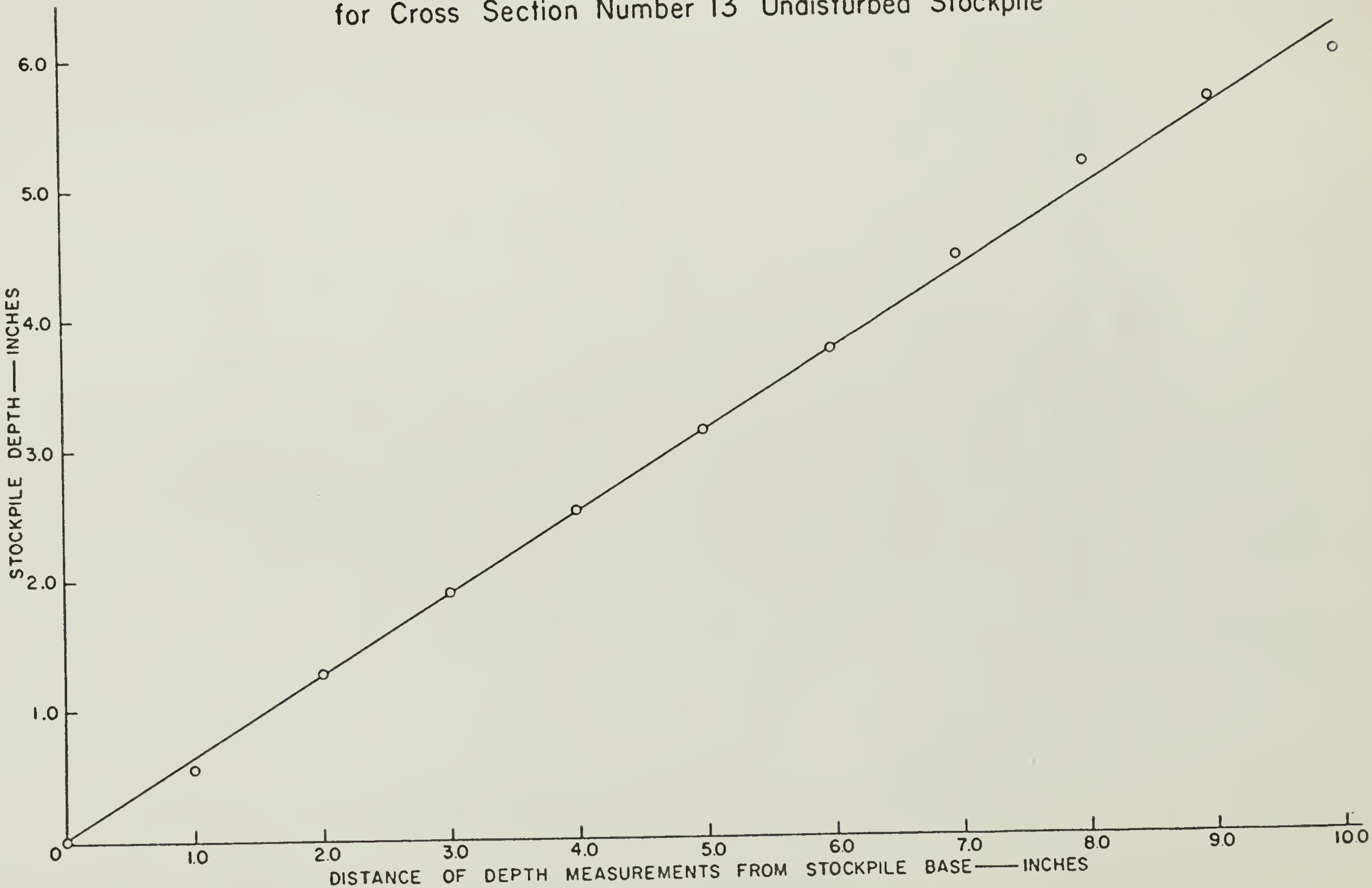
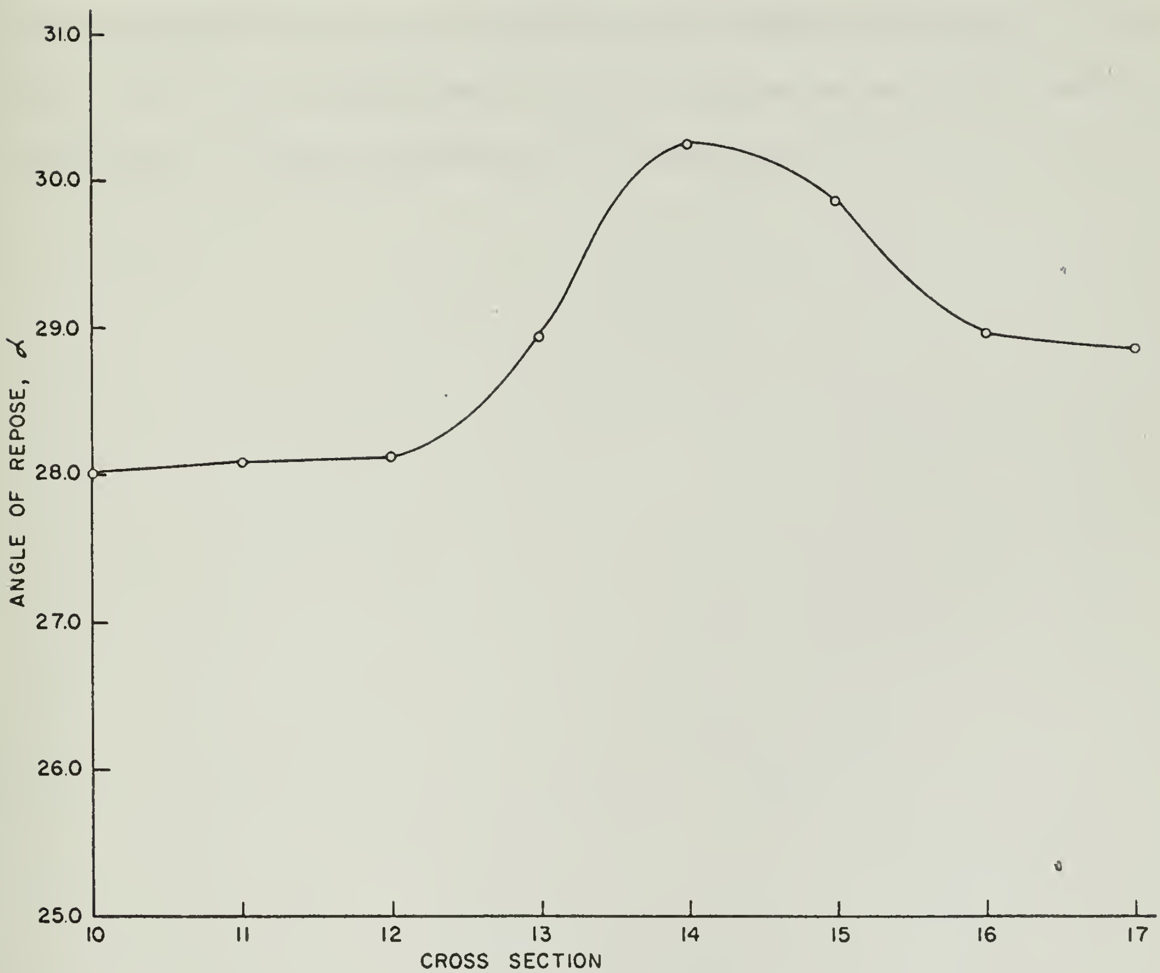


Figure 4.9

Angle of Repose for Disturbed Stockpile



it can be seen that it is now possible to have adjacent angles of repose varying from 28.0 to 30.2 degrees for a disturbed stockpile, the average being approximately 29.5 degrees.

One may think of the 31.6 degrees angle of repose for an undisturbed stockpile as the initial condition of a stockpile. With the passage of time, factors such as weight of the granules, vibration, moisture, and size of granules cause the stockpile to compact and thereby reduce the angle of repose. For this experiment lasting approximately four weeks, the angle of repose decreased 2.1 degrees.

CHAPTER V

APPLICATION OF RESULTS

General

This chapter discusses the principles and methods of volume estimation for commercial scale stockpiles based on the methods developed in the previous chapters for laboratory scale stockpiles. The use of a digital computer program and a manual computation method utilizing the stockpile base measurements and the apparent angle of repose is explained. The computer program, Model #1, given in Appendix A, is adopted as the most appropriate mathematical model to be used in estimating the volume of stockpiles. This mathematical model, which assumes that the typical stockpile cross section is an isosceles triangle, is the basis for the development of the manual computation forms shown in Appendix B. Appendix B contains one blank manual computation form and one completed sample calculation.

Reserves Estimation Using a Digital Computer

In using Model #1 computer program to compute the volumes of commercial scale stockpiles, it is first necessary to establish a boundary system in the stockpile warehouse. The boundary system can be either painted on the floor or associated with the stanchions of the warehouse. The side boundaries should be located as close to the base of the stockpile as is physically possible. Also, the end boundaries

should be selected after proper recognition is taken of the number of cross sections to be used per peak. The total number of cross sections taken should be an odd number. Further, although not absolutely necessary, it would be wise to select the interval between cross sections so that the cross sections all fall on marked scale divisions of the base grid. These criteria allow a maximum flexibility in the use of the Trapezoid and Simpson's integration formulas, and maximum simplicity in making the measurements.

In determining the number of cross section base measurements to be taken, a decision needs to be made as to whether the stockpile is classified as "undisturbed" or "disturbed". If the stockpile is undisturbed, the general rule is to take at least three base measurements per each peak on the stockpile. The resultant error in the estimated volume will be within approximately $\pm$ 3%. If more than 3 measurements per peak are taken the resultant error in the estimated volume can be as low as $\pm$ 1%.

On the other hand, if the stockpile is disturbed, the general rule is to take at least 5 base measurements per each peak. The error in the estimated volume will be approximately in the range of $\pm$ 3%. If more than 5 measurements per peak are taken the error in the estimated volume will approximate $\pm$ 1%.

Before a final selection is made as to the number of measurements to be taken, consideration should be given to the rough estimate of the amount of material in the stockpile.

Good inventory management practice dictates that the larger the stockpile, the more accurate should be the volume determination. Thus, the maximum number of base measurements should be taken consistent with the labor costs necessary to take the measurements.

Once the number of base measurements to be taken has been determined, the next step is to begin measuring the distance between the base of the stockpile and the side boundary line. The measurements should be to an accuracy of 1/10 feet or the nearest inch. The cross section base measurements are required to be equally spaced over the length of the boundary grid and should of course be taken on both sides of the stockpile. The measurements should be recorded on a data sheet similar to that shown in Figure 1, Appendix B.

Upon completion of the base measurements, the volume and tonnage estimates for the stockpile can be computed with a digital computer. The measurement data should be keypunched following the instructions given in Appendix A for the Model #1 computer program. The computer program also requires the length and width of the boundary grid together with the apparent angle of repose to be read in as data. The apparent angle of repose must be determined for different kinds of stockpile material in the manner outlined in Chapter III. The proper formats for all data cards are prescribed in the computer program.

The computer program computes the estimated volume in the units selected for the input data.

The weight of material in the stockpile is then calculated using the average bulk density of the material.

Reserves Estimation by Manual Calculation

Appendix B contains both a blank manual computation form, Figure 1, and a completed manual computational form, Figure 2. This form was designed to use the same measurements as described for the computer program. The form is designed in three basic parts. At the top of the form is a block for constants. These are self explanatory and are used in the remaining two parts for computing the cross sectional areas, volume, and weight of stockpile material. The center section of the form calculates the areas of the cross sections utilizing the base measurement distances in Columns 2 and 3 and assuming that a typical cross section is an isosceles triangle. Columns 7, 8, and 9 of the form are totaled and these results form the basis for the volume and tonnage determination of the stockpile.

In the computation form, provision is made to compute the reserves by two different means; one using the Trapezoid Rule, the other using Simpson's Rule for the lengthwise integration of cross section areas. In general, these will give close but not identical answers as in the present case. It is suggested that either the two answers be averaged, or the most conservative estimate be selected and used for inventory management purposes.

CHAPTER VI

SUMMARY AND RECOMMENDATIONS

In this thesis an attempt has been made to improve the volume estimation of commercial stockpiles through the judicious use of mensuration and numerical integration methods on laboratory scale stockpiles.

During this study, the basic principle of mensuration was kept foremost in mind, namely, to determine the area of surfaces or the volume of solids from certain simple data on lines and angles.

Using numerical analysis methods, it was found to be advantageous to unite the two computational methods. The experimental results show that the areas of all cross sections of a stockpile can be readily and accurately approximated by an isosceles triangle. Moreover, the area of an isosceles triangle can be determined by two simple measurements, its base length and angle of inclination. The application of either the Trapezoid or Simpson Rule of numerical integration to the approximate cross sectional areas results in the most accurate volume estimates. The choice of the rule is somewhat arbitrary but depends essentially on the degree of regularity of the stock pile surface. It is recommended that the Trapezoid Rule be used for disturbed surfaces and the Simpson Rule for undisturbed surfaces, i.e., those formed by pouring material directly above the pile. Thus, advantage can be taken of both mensuration and

numerical integration concepts to satisfy the requirement for a minimum number of measurements to attain a relative high accuracy for the volume determination.

The study has shown that, for a laboratory scale stockpile, this method of calculating the volume is realistic. It is believed that the results of the present laboratory measurements can be adapted to the dimensions of commercial stockpiles by considering the number of cross section measurements taken per stockpile peak. For practical use, 5 cross sections per peak are sufficient although 3 cross sections per peak generally give errors of less than 10%.

Finally, if this technique has awakened engineers and businessmen to the prospects of making a better determination of materials stored in a stockpile, for inventory management purposes, the main purpose of this thesis has been accomplished.

NOTES

1. Homer A. Black and John E. Champion, Accounting in Business Decision, (Englewood Cliffs, N. J., Prentice-Hall, Inc., 1962), p. 440.

2. IBID., p. 472.

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5. J. B. Scarborough, Numerical Mathematical Analysis, (Baltimore, The Johns Hopkins Press, 1958), p. 131.

6. IBID., pp. 51-127.

7. IBID., p. 61.

8. Leon Lapidus, Digital Computation for Chemical Engineers, (New York, McGraw-Hill Book Company, Inc., 1962), pp. 13-81.

9. W. A. Granville, P. F. Smith and W. R. Longley, Elements of Calculus, (Boston, The Athenaeum Press, Ginn and Company, 1946), pp. 113-167.

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13. Scarborough, J. B., Numerical Mathematical Analysis, Fifth Edition, The Johns Hopkins Press, Baltimore, 1962.
14. U. S. Dept. of Commerce, National Bureau of Standards, Handbook of Mathematical Functions, Applied Mathematics Series No. 55, U. S. Government Printing Office, Washington, 1964.

APPENDIX A

COMPUTER PROGRAM TO DETERMINE THE APPROXIMATE VOLUME OF A STOCKPILE ASSUMING THE STOCKPILE CROSS SECTIONS ARE ISOSCELES TRIANGLES.

Purpose

The purpose of this program is to determine the approximate volume of a stockpile by numerical integration assuming isosceles triangle cross sections. A secondary purpose is to determine the angle of repose of the material forming the stockpile.

LANGUAGE

Fortran IV (IBM 7040 Computer)

SYMBOLIC DICTIONARY

VARIABLE	S/A*	I/O**	DESCRIPTION
XR	A	I	Distance the base of the pile is from right boundary line.
XL	A	I	Distance the base of the pile is from left boundary line.
M	S	I/O	Number of measurement points.
L	S	I/O	Distance between the end boundary lines.
W	S	I/O	Distance between the side boundary lines.

*S - Single variable; A - Array of variables

**I - Input; O - Output

ALPHA	S	I/O	Inclination angle of the stockpile - same as apparent angle of repose.
ALPHAR	S	--	Alpha expressed in radians.
N	S	I	Number of measurement stations.
VACT	S	I/O	Actual volume of the stockpile.
DELTAL	S	O	Length interval size.
BASE	S	--	Width of stockpile at the base.
AREA	A	O	Area of a cross section.
SUMA	S	--	A holding variable used to accumulate the areas of cross sections for calculating the volume by the Trapezoidal Rule.
VCALTR	S	O	The calculated volume of the stockpile utilizing the Trapezoidal Rule.
PERRTR	S	O	Per cent error of the calculated volume using the Trapezoidal Rule.
ODAREA	S	--	A holding variable used to accumulate the areas of the odd numbered stockpile cross sections for calculating the stockpile volume using Simpson's Rule.
EVAREA	S	--	A holding variable used to accumulate the areas of the even numbered cross sections for calculating the stockpile volume using Simpson's Rule.
VCALSN	S	O	The calculated stockpile volume using Simpson's Rule.
PERRSN	S	O	Per cent error of the calculated volume using Simpson's Rule.

SUMTWA	S	--	A holding variable used to accumulate the weighted areas of cross sections for calculating the stockpile volume using Weddle's Rule.
VCALWD	S	O	The calculated stockpile volume using Weddle's Rule.
PERRWD	S	O	Per cent error of the calculated volume using Weddle's Rule.

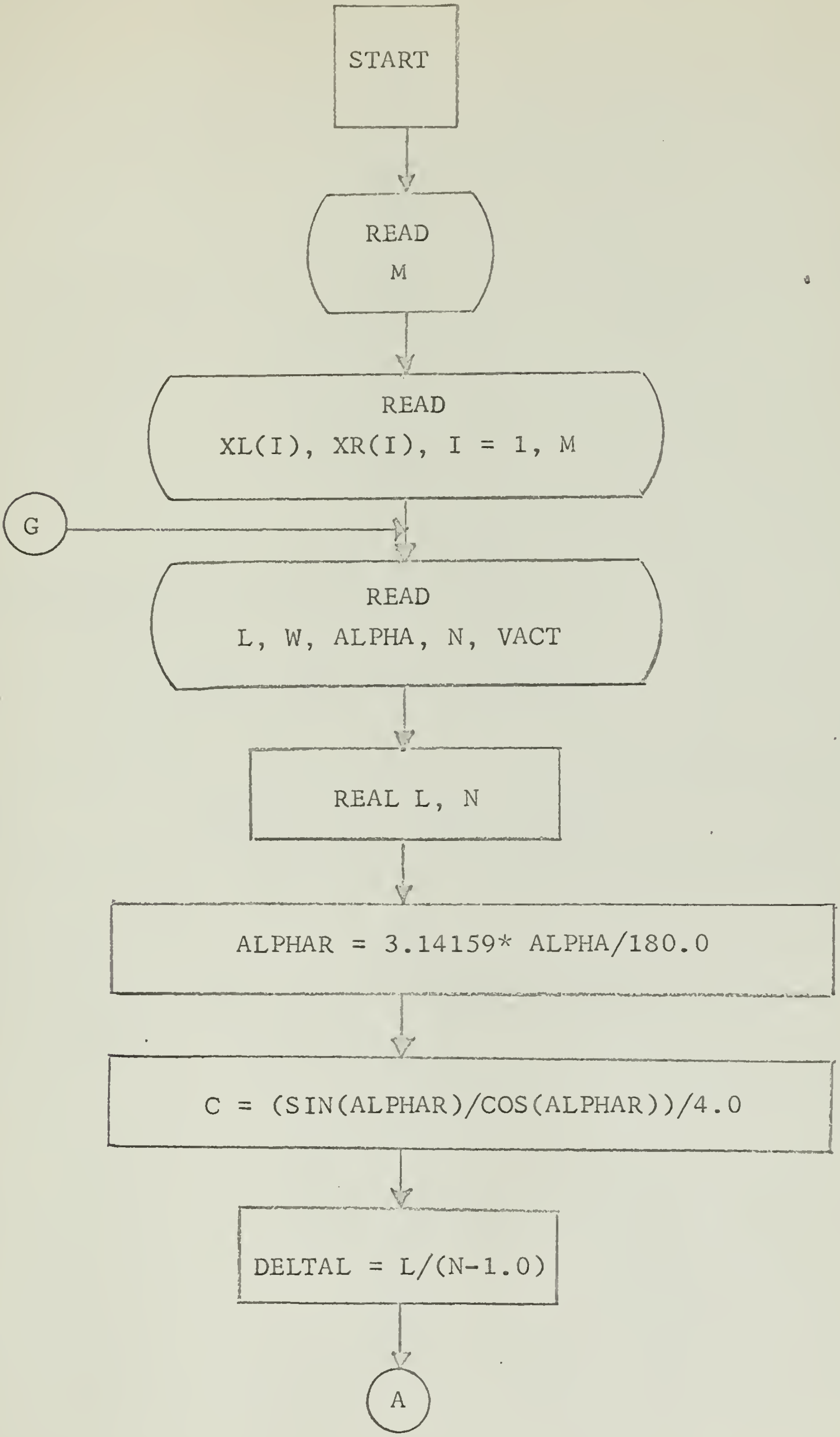
PROGRAM ROUTINE

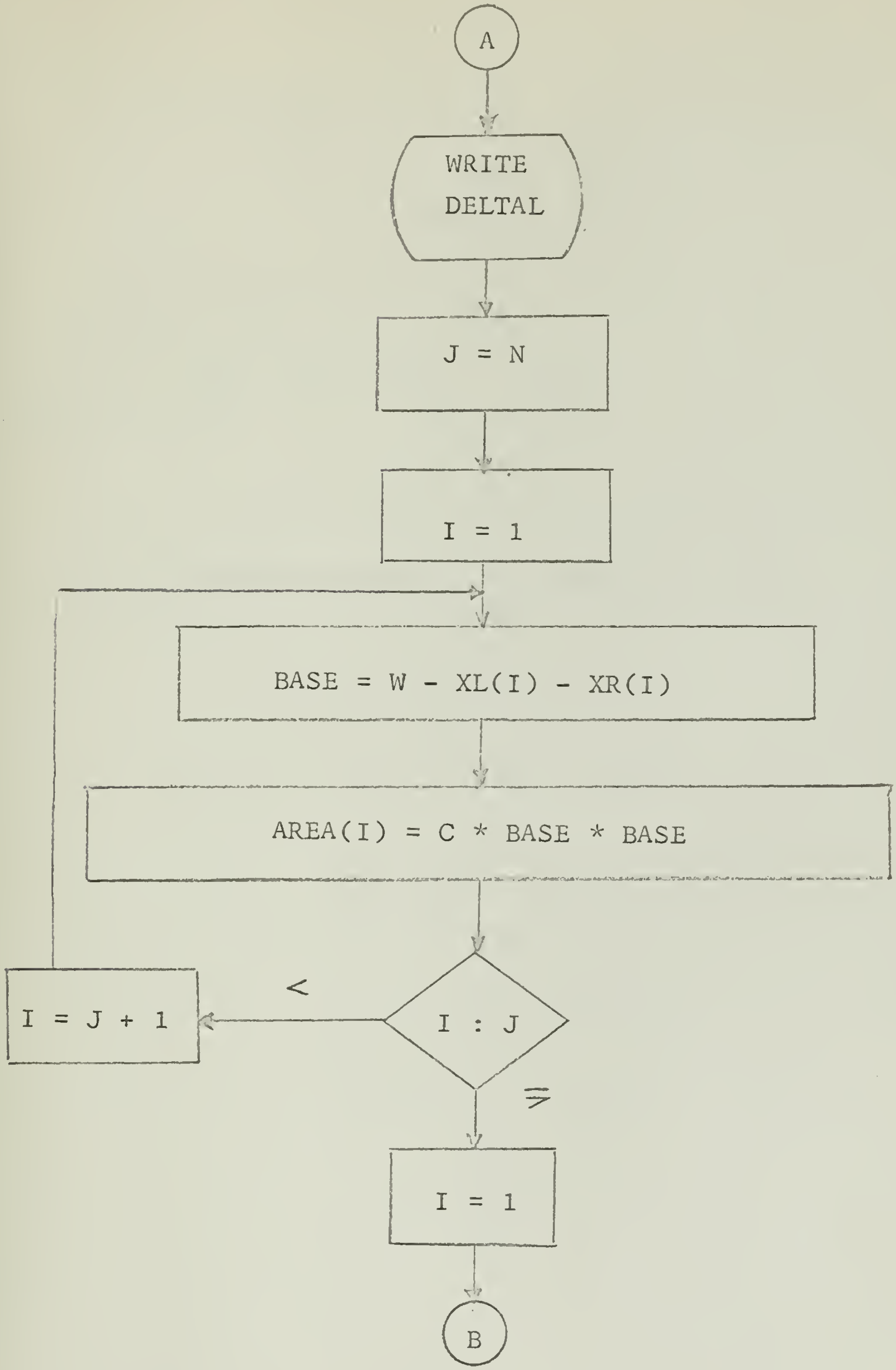
This program utilizes the data points (representing the distances to the side boundary lines) for each cross section. The area of each cross section is then computed using the angle of repose as a parameter. These areas are then integrated by the Trapezoid, Simpson's and Weddle's quadrature rules for a set interval size to obtain the calculated volume of the stockpile. The per cent error in the calculated volume is then computed. The angle of repose is determined when the per cent error is zero.

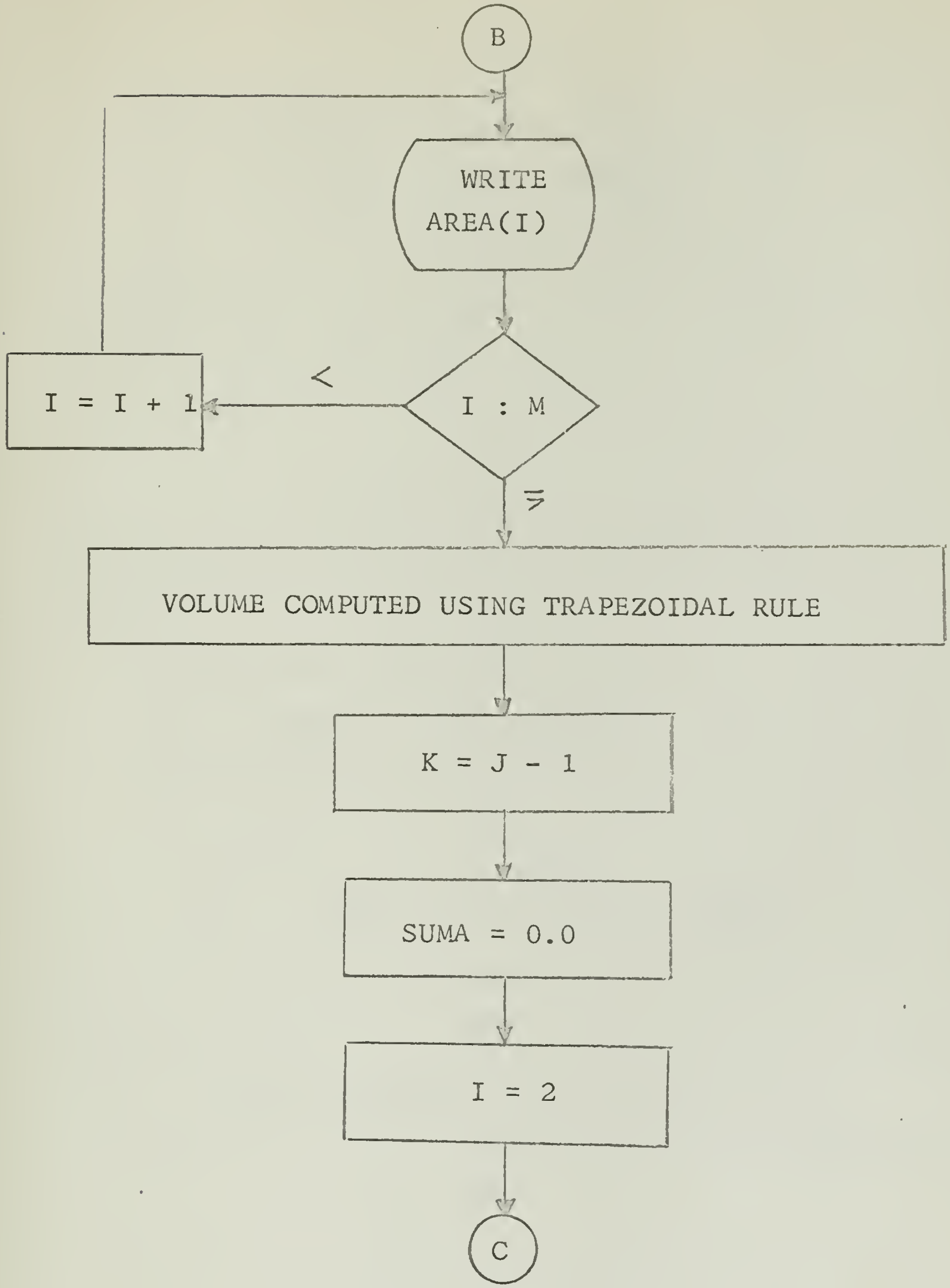
```
C     THIS PROGRAM DETERMINES THE APPROXIMATE VOLUME OF A STOCKPILE
C     ASSUMING THAT THE CROSS SECTIONS ARE ISOCELES TRIANGLES.
C
      DIMENSION XL(100) , XR(100) , AREA(100)
      READ (5,5)  M
    5 FORMAT (I2)
      WRITE ( 6,6)
    6 FORMAT (15X, 20HM = NO. OF DATA PTS.//)
      WRITE (6,7)  M
    7 FORMAT (19X, I2//)
      READ (5,20) (XL(I) , XR(I) , I = 1, M)
   20 FORMAT (2F10.2)
      WRITE (6,11)
   11 FORMAT (15X, 17HINPUT DATA POINTS//)
      WRITE (6,13)
   13 FORMAT (15X, 3HNO., 15X, 2HXL, 15X, 2HXR//)
      WRITE (6,12) ( I, XL(I), XR(I) , I = 1, M)
   12 FORMAT ( 15X, I2, 10X, F10.2,  8X, F10.2//)
    9 READ (5,10) L, W, ALPHA, N, VACT
   10 FORMAT (2F10.2,F6.4, F6.0, F10.2)
      WRITE (6,14)
   14 FORMAT (15X, 20HINPUT DATA CONSTANTS//)
      WRITE (6,16)
   16 FORMAT(15X, 1HL, 10X, 1HW, 10X, 5HALPHA, 10X, 1HN, 10X, 4HVACT/
      WRITE (6,17) L, W, ALPHA, N, VACT
   17 FORMAT(13X, F5.2, 6X, F5.2, 8X, F5.2, 8X, F5.0, 9X, F10.2//)
C
C     ASSUME CROSS SECTIONS ARE ISOCELES TRIANGLES FOR COMPUTING AREA
      REAL L,N
      ALPHAR = 3.14159 * ALPHA / 180.0
      C = (SIN(ALPHAR)/COS(ALPHAR))/4.0
      DELTAL = L/(N-1.0)
      WRITE (6,18)
   18 FORMAT (15X, 17HINTERVAL = DELTAL//)
      WRITE (6,19)  DELTAL
   19 FORMAT (22X, F5.2//)
      J = N
      DO 100 I = 1, J
      BASE = W - XL(I) - XR(I)
  100 AREA(I) =  C *BASE *BASE
      WRITE (6,500)
  500 FORMAT (15X, 22HAREA OF CROSS SECTIONS//)
      WRITE (6,600) (I, AREA(I), I = 1, M)
  600 FORMAT(15X, I2, F10.4)
C
C     STOCKPILE VOLUME COMPUTED WITH THE TRAPEZOID INTEGRATION RULE.
      K = J - 1
      SUMA = 0.0
      DO 800 I = 2, K
  800 SUMA = SUMA + AREA(I)
      VCALTR = (DELTAL/2.0) * (AREA(1) + 2.0 *SUMA + AREA(M))
      PERRTR = 100.0 * (VCALTR - VACT) / VACT
      WRITE ( 6,900)
  900 FORMAT(15X, 5HALPHA, 15X, 4HVACT, 15X, 6HVCALTR, 15X, 6HPERRTR/
      WRITE (6,400) ALPHA, VACT,  VCALTR,  PERRTR
```

```
  400 FORMAT (15X, F5.2, 15X, F7.2, 12X, F7.2, 13X, F6.2//)
C
C     STOCKPILE VOLUME COMPUTED WITHSIMPSONS INTEGRATION RULE.
      KK = J - 1
      KKK = KK - 1
      ODAREA = 0.0
      EVAREA = 0.0
      DO 910 I = 2, KK, 2
  910 EVAREA = EVAREA + AREA (I)
      DO 920 I = 3, KKK, 2
  920 ODAREA = ODAREA + AREA (I)
      VCALSN = (DELTAL/3.)*(AREA(1) + 4.*EVAREA + 2.*ODAREA + AREA(M))
      PERRSN = 100.0 * (VCALSN - VACT) / VACT
      WRITE ( 6, 950)
  950 FORMAT(15X, 5HALPHA, 15X, 4HVACT, 15X, 6HVCALSN, 15X, 6HPERRSN//
      WRITE (6,400)  ALPHA,  VACT,  VCALSN,  PERRSN
C
C     STOCKPILE VOLUME COMPUTED WITHWEDDLES INTEGRATION RULE.
      SUMTWA = 0.0
      DO 960  I = 2, M, 6
  960 SUMTWA = SUMTWA + 1.*AREA(I-1) + 5.*AREA(I) + 1.*AREA(I+1) +
     16.*AREA(I+2) + 1.*AREA(I+3) + 5.*AREA(I+4) + 1.*AREA(I+5)
      VCALWD = .3*DELTAL*SUMTWA
      PERRWD = 100.0*(VCALWD - VACT)/VACT
      WRITE (6,970)
  970 FORMAT(15X, 5HALPHA, 15X, 4HVACT, 15X, 6HVCALWD, 15X, 6HPERRWD/
      WRITE (6,400) ALPHA, VACT, VCALWD, PERRWD
      GO TO 9
      CALL EXIT
      END
```

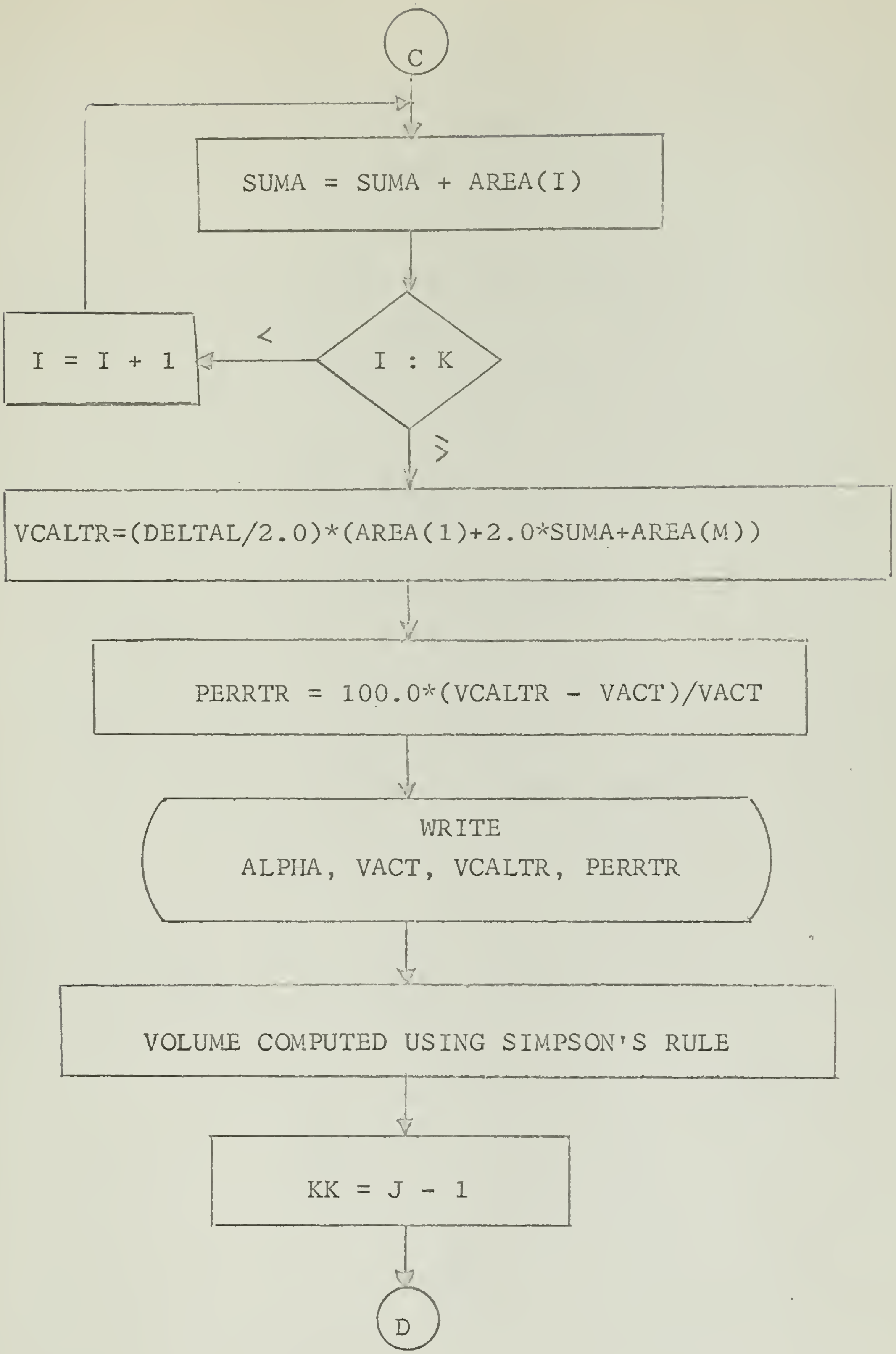
FLOW DIAGRAM FOR COMPUTER PROGRAM TO DETERMINE THE APPROXIMATE VOLUME OF A STOCKPILE ASSUMING THE CROSS SECTIONS TO BE ISOSCELES TRIANGLES.

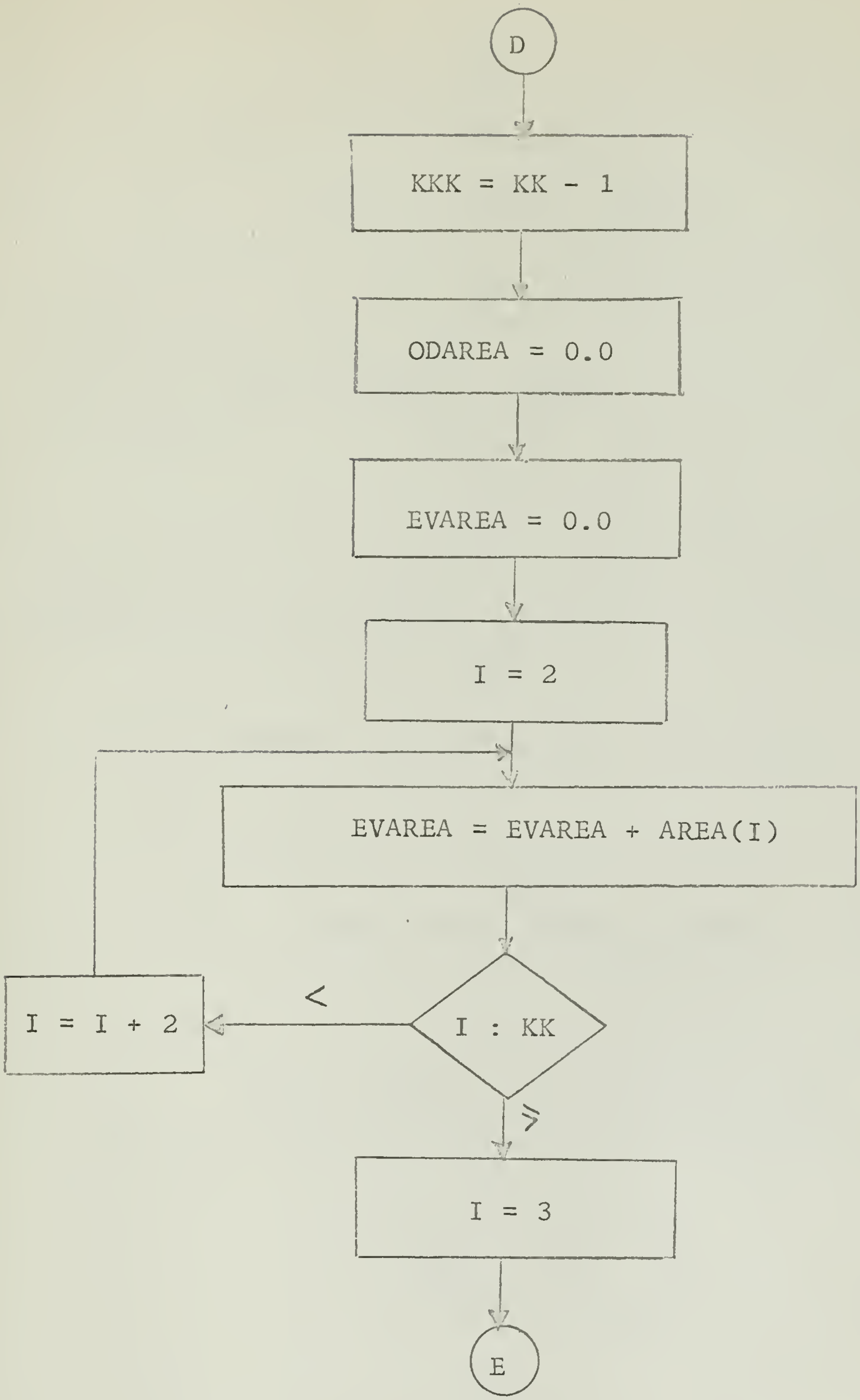


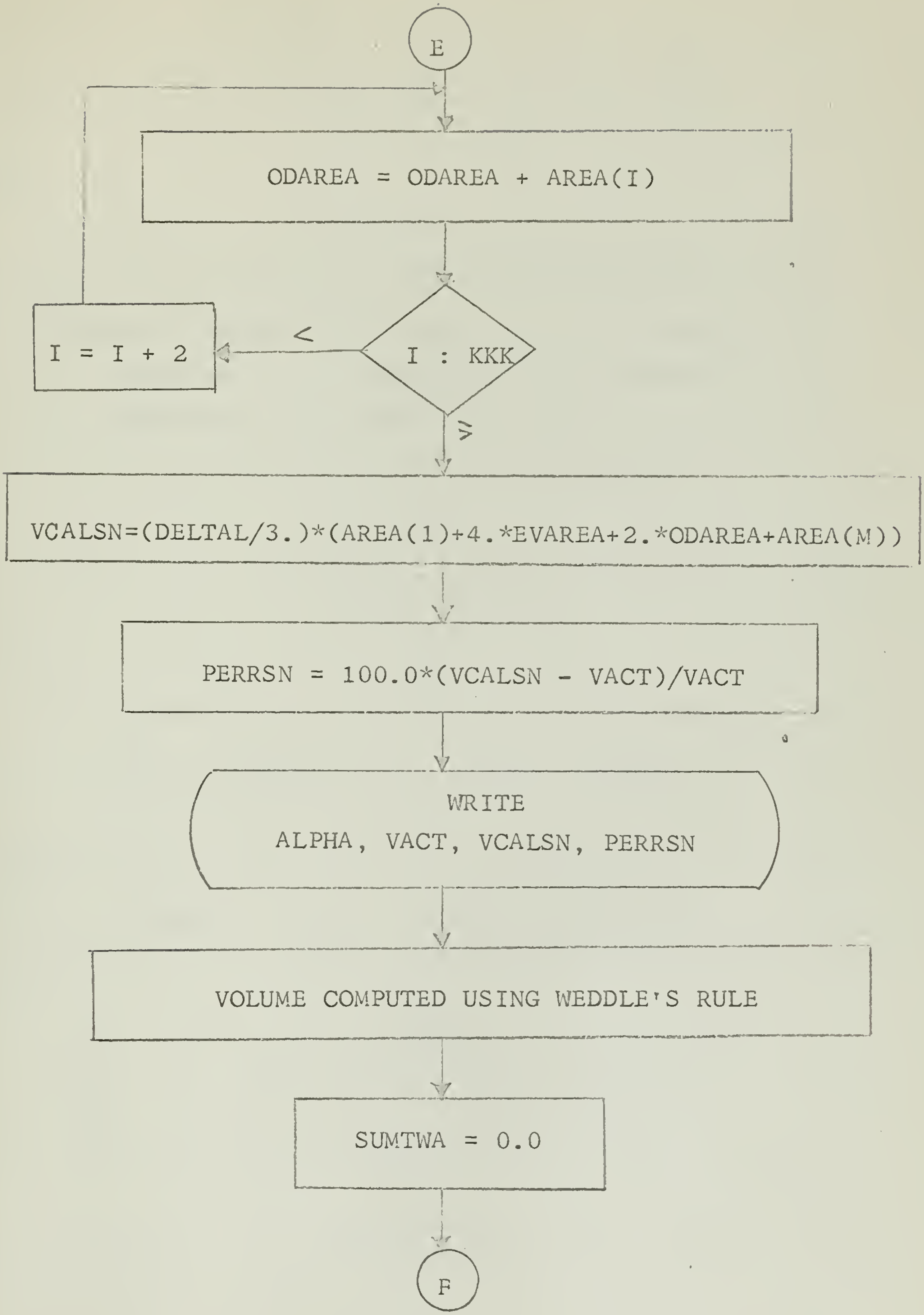

A
WRITE DELTAL
J = N
I = 1
BASE = W - XL(I) - XR(I)
AREA(I) = C * BASE * BASE
I : J
<
I = J + 1
≥
I = 1
B

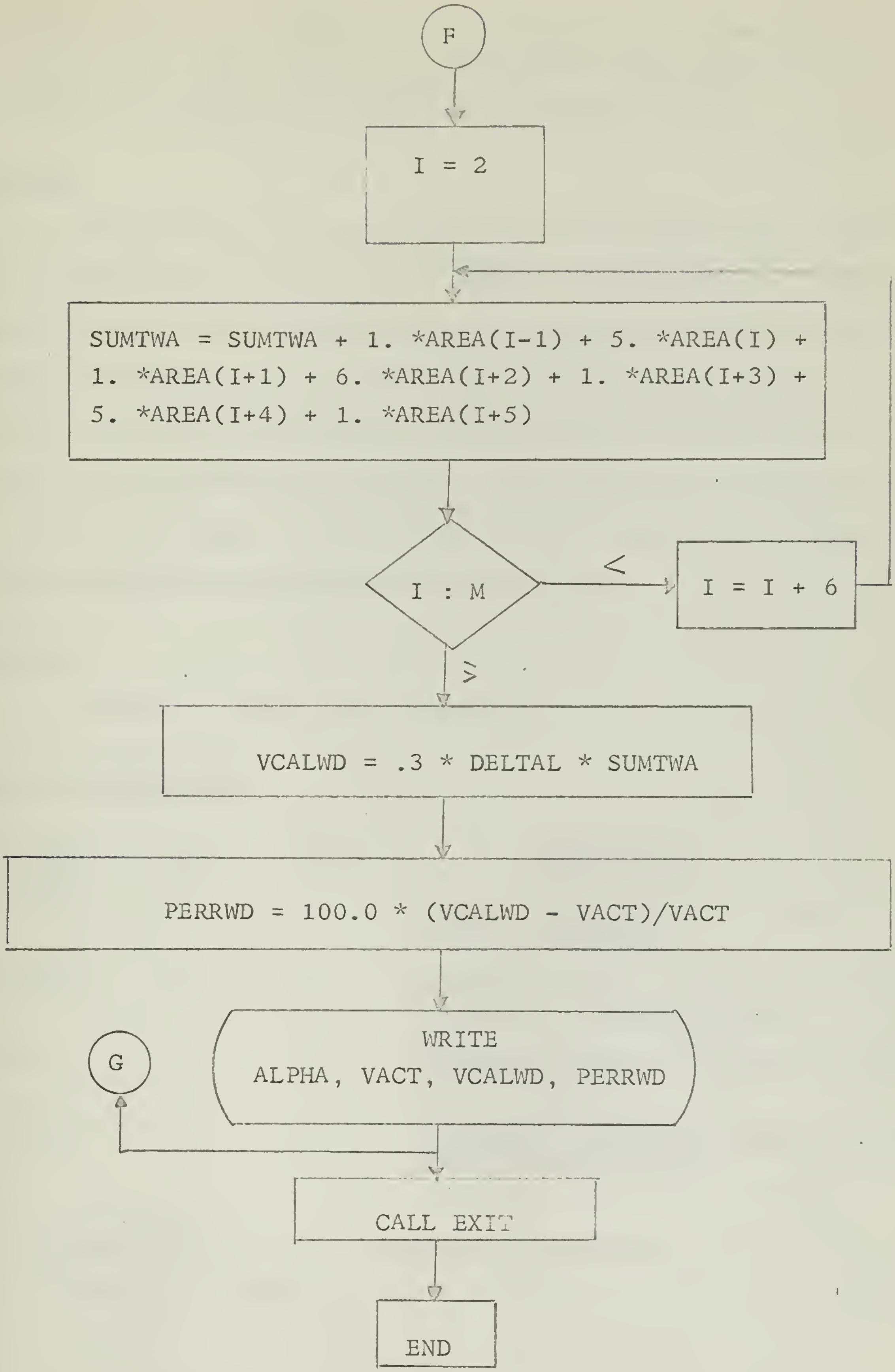


B
WRITE AREA(I)
I : M
<
≥
I = I + 1
VOLUME COMPUTED USING TRAPEZOIDAL RULE
K = J - 1
SUMA = 0.0
I = 2
C


C
SUMA = SUMA + AREA(I)
I = I + 1
<
I : K
≥
VCALTR=(DELTAL/2.0)*(AREA(1)+2.0*SUMA+AREA(M))
PERRTR = 100.0*(VCALTR - VACT)/VACT
WRITE
ALPHA, VACT, VCALTR, PERRTR
VOLUME COMPUTED USING SIMPSON'S RULE
KK = J - 1
D


D
KKK = KK - 1
ODAREA = 0.0
EVAREA = 0.0
I = 2
EVAREA = EVAREA + AREA(I)
I : KK
<
I = I + 2
≥
I = 3
E


E
ODAREA = ODAREA + AREA(I)
I = I + 2
<
I : KKK
≥
VCALSN=(DELTAL/3.)*(AREA(1)+4.*EVAREA+2.*ODAREA+AREA(M))
PERRSN = 100.0*(VCALSN - VACT)/VACT
WRITE
ALPHA, VACT, VCALSN, PERRSN
VOLUME COMPUTED USING WEDDLE'S RULE
SUMTWA = 0.0
F


F
I = 2
SUMTWA = SUMTWA + 1. *AREA(I-1) + 5. *AREA(I) + 1. *AREA(I+1) + 6. *AREA(I+2) + 1. *AREA(I+3) + 5. *AREA(I+4) + 1. *AREA(I+5)
I : M
<
I = I + 6
≥
VCALWD = .3 * DELTAL * SUMTWA
PERRWD = 100.0 * (VCALWD - VACT)/VACT
WRITE
ALPHA, VACT, VCALWD, PERRWD
G
CALL EXIT
END

COMPUTER PROGRAM TO DETERMINE THE APPROXIMATE VOLUME OF A STOCKPILE BY DOUBLE NUMERICAL INTEGRATION UTILIZING THE TRAPEZOID, SIMPSON'S AND WEDDLE'S QUADRATURE RULES.

Purpose

The purpose of this program is to determine the approximate volume of a stockpile by numerical integration of the height measurements for each cross section using the Trapezoidal, Simpson's and Weddle's Rule. Then, using these results, determine the volume of the stockpile through the use of the three quadrature formulas. Throughout the calculations, the number of height measurements and the number of stockpile cross sections are designated as parameters.

LANGUAGE

Fortran IV (IBM 7040 Computer)

SYMBOLIC DICTIONARY

VARIABLE	S/A*	I/O**	DESCRIPTION
CRSECT	A	I	Height measurements in feet of the stockpile.
AREATR	A	O	Computed area of cross sections using the Trapezoid Rule.
AREASN	A	O	Computed area of a cross section using Simpson's Rule.
AREAWD	A	O	Computed area of a cross section using Weddle's Rule.

*S - Single Variable; A - Array of Variables

**I - Input; O - Output

DIFF 1	A	O	Difference in area computed by the Trapezoidal Rule and Simpson's Rule.
DIFF 2	A	O	Difference in area computed by the Trapezoidal Rule and Weddle's Rule.
DIFF 3	A	O	Difference in area computed by Simpson's Rule and Weddle's Rule.
NPW	S	I	Maximum number of cross section height measurements.
NPL	S	I	Maximum number of cross sectional areas for the stockpile.
NPWX	S	I/O	Actual number of cross section height measurements for a specific calculation.
NPLX	S	I/O	Actual number of cross sectional areas for a specific calculation.
L	S	I/O	Distance between the end boundary lines.
W	S	I/O	Distance between the side boundary lines.
VACT	S	I/O	Actual volume of the stockpile.
DELTAW	S	O	Cross section interval size.
DELTAL	S	O	Length interval size.
SUMPTS	S	--	A holding variable used to accumulate the height measurements for calculating the areas of cross sections with both the Trapezoid Rule and Weddle's Rule.
ODDPTS	S	--	A holding variable used to accumulate the odd numbered height measurements for calculating the areas of cross sections with Simpson's Rule.
EVNPTS	S	--	A holding variable used to accumulate the even numbered height measurements for calculating the areas of cross sections with Simpson's Rule.

SUMA	S	--	A holding variable used to accumulate the areas of cross sections for calculating the volume of the stockpile utilizing the Trapezoidal Rule and Weddle's Rule.
ODAREA	S	--	A holding variable used to accumulate the odd numbered areas of cross sections for calculating the volume of the stockpile utilizing Simpson's Rule.
EVAREA	S	--	A holding variable used to accumulate the even numbered areas of cross sections for calculating the volume of the stockpile utilizing Simpson's Rule.
VCTRTR	S	O	The volume of the stockpile computed by numerical integration of the cross section height measurements by the Trapezoid Rule and then numerically integrating the resulting cross sectional areas by the Trapezoid Rule to obtain the volume of the stockpile.
VCTRSN	S	O	Volume computed using Trapezoid Rule and Simpson's Rule.
VCTRWD	S	O	Volume computed using Trapezoid Rule and Weddle's Rule.
VCSNTR	S	O	Volume computed using Simpson's Rule and Trapezoid Rule.
VCSNSN	S	O	Volume computed using Simpson's Rule twice.
VCSNWD	S	O	Volume computed using Simpson's Rule and Weddle's Rule
VCWDTR	S	O	Volume computed using Weddle's Rule and Trapezoid Rule.
VCWDSN	S	O	Volume computed using Weddle's Rule and Simpson's Rule.
VCWDWD	S	O	Volume computed using Weddle's Rule twice.

PETRTR	S	O	The per cent error in the calculated volume when the calculated volume is obtained by numerically integrating the cross section height measurements with the Trapezoid Rule and subsequently numerically integrating the resulting cross sectional areas using the Trapezoid Rule.
PETRSN	S	O	The per cent error in the calculated volume utilizing Trapezoid and Weddle's Rules.
PESNTR	S	O	The per cent error in the calculated volume utilizing Simpson's and Trapezoidal Rules.
PESNSN	S	O	The per cent error in calculated volume utilizing Simpson's Rule twice.
PEWDTR	S	O	The per cent error in calculated volume utilizing Weddle's and Trapezoidal Rules.
PEWDSN	S	O	The per cent error in calculated volume utilizing Weddle's and Simpson's Rules.
PEWDWD	S	O	The per cent error in calculated volume utilizing Weddle's Rule twice.

PROGRAM ROUTINE

This program utilizes as data the height measurements, at one inch intervals, of the stockpile cross sections. The area of each cross section is then computed utilizing all three quadrature formulas while varying the number of height measurements from the maximum to the minimum.

The resultant areas are then numerically integrated by three quadrature formulas to compute the volume of the stockpile. The number of cross sectional areas is varied from the maximum to the minimum during this phase of the computations.

The resulting nine combinations for calculating the stockpile volume are then compared to the known volume to obtain the per cent error in the calculated volume.

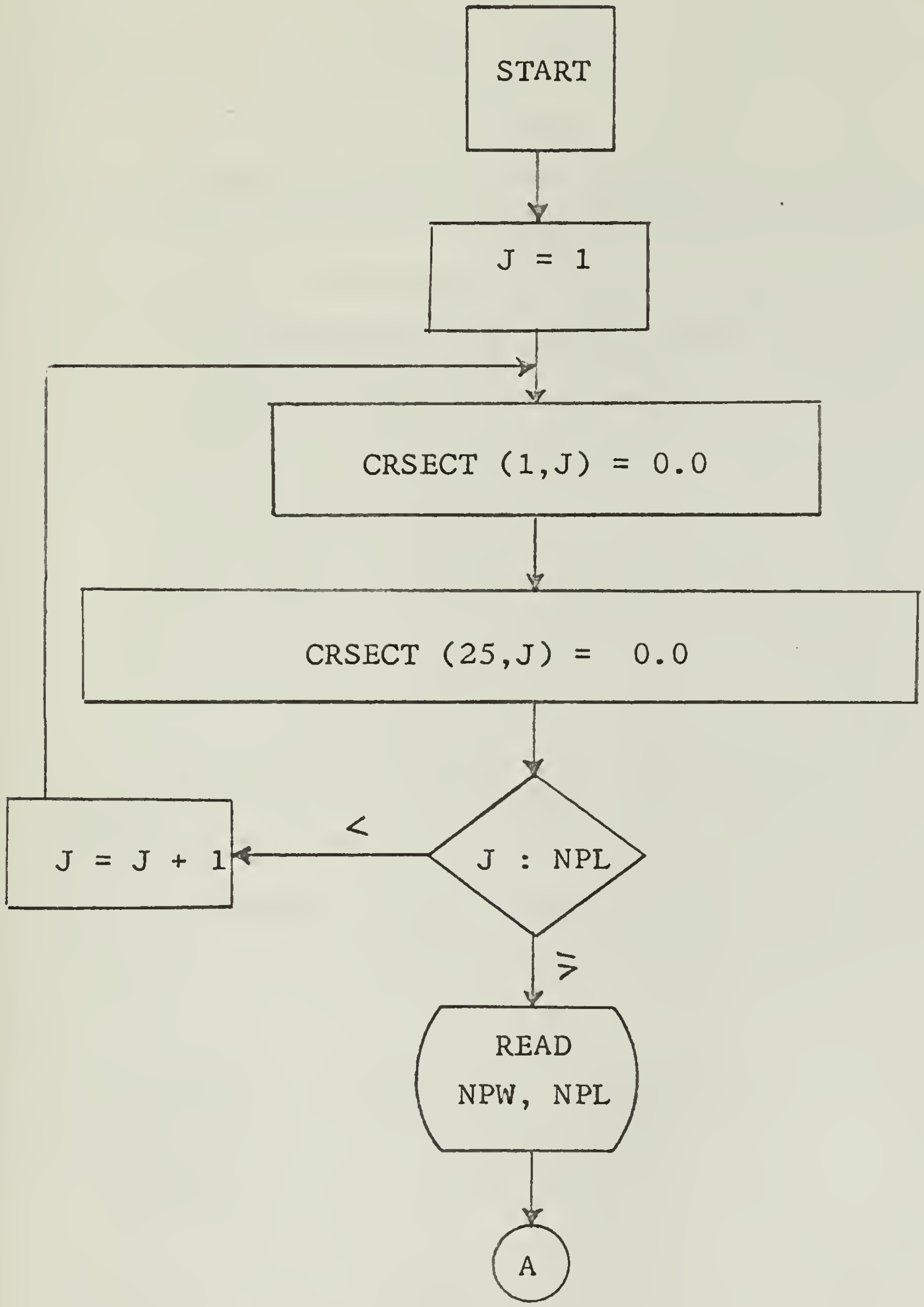
```
C     THIS PROGRAM CALCULATES THE VOLUME OF AN IRREGULAR STOCKPILE BY
C     DOUBLE INTEGRATION USING THE TRAPEZOIDAL, SIMPSONS, AND
C     WEDDLES RULES.
C
      DIMENSION CRSECT(30,70), AREATR(70), AREASN(70), AREAWD(70)
      DIMENSION  DIFF1(70), DIFF2(70) , DIFF3(70)
      READ (5.5) NPW, NPL
    5 FORMAT (2I3)
      WRITE (6,4)
    4 FORMAT (1H1)
      READ (5,26)  NPWX, NPLX
   26 FORMAT (2I3)
      WRITE (6,6)
    6 FORMAT(15X,22HNO. OF CRSECT DATA PTS,5X,22HNO. OF LENGTH DATA PTS)
      WRITE (6,7)   NPWX ,  NPLX
    7 FORMAT (25X, I3, 22X, I3//)
      READ(5,10) ((CRSECT(I,J), I=1, NPW), J=1, NPL)
   10 FORMAT (16F5.3/7F5.3)
C
C     CONVERTING DATA VALUES INTO INCHES
      DO 11 J = 1, NPL
      DO 11 I = 1, NPW
   11 CRSECT(I,J) = CRSECT(I,J)*12.0
      READ (5,20)  L, W, VACT
   20 FORMAT (2I3, F10.2)
      WRITE (6,21)
   21 FORMAT (15X, 20HINPUT DATA CONSTANTS//)
      WRITE (6,22)
   22 FORMAT (15X, 1HL, 15X, 1HW, 15X, 4HVACT//)
      WRITE (6,23)  L,  W,  VACT
   23 FORMAT (15X, I3, 12X, I3, 8X, F10.2//)
      INTEGER  W
      DELTAW = W/(NPWX - 1)
      DELTAL = L/(NPLX - 1)
      WRITE (6,24)  DELTAW,  DELTAL
   24 FORMAT(15X, 9HDELTAW = , F5.2, 5X, 9HDELTAL = , F5.2//)
C
C     AREA OF CROSS SECTIONS COMPUTED BY TRAPEZOIDAL, SIMPSONS
C     AND WEDDLES RULES
C
C     1ST, TRAPEZOID RULE
C
      K = NPW - 1
      DO 30 J = 1, NPL
      SUMPTS = 0.0
      DO 31 I = 2, K
   31 SUMPTS = SUMPTS + CRSECT(I,J)
   30 AREATR(J)=(DELTAW/2.)*(CRSECT(1,J)+2.*SUMPTS+CRSECT(NPW,J))
C
C     2ND, SIMPSONS RULE
C
      KK = NPW - 1
      KKK = KK - 1
      DO 32 J = 1, NPL
      ODDPTS = 0.0
      EVNPTS = 0.0
```

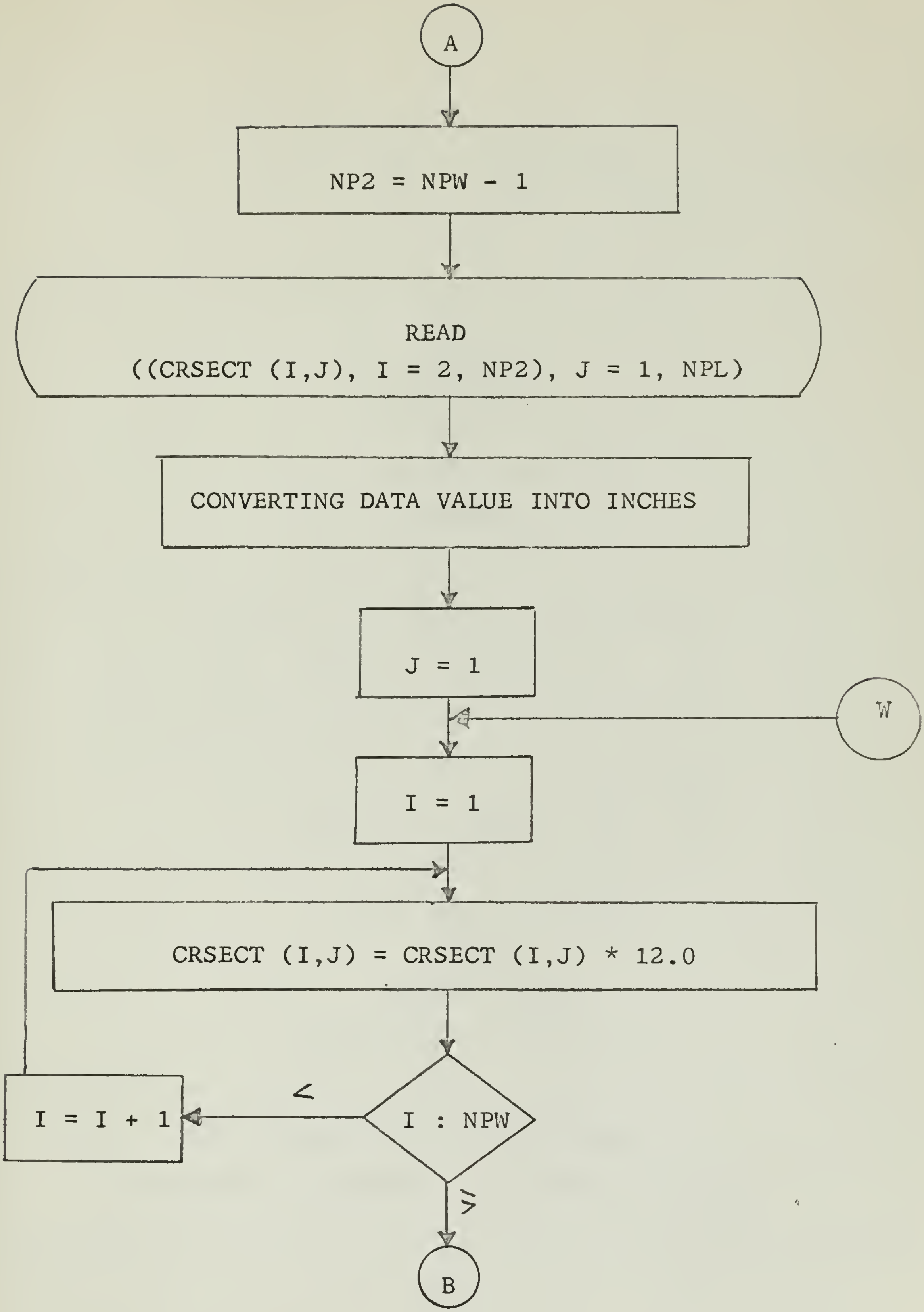
```
      DO 33 I = 2, KK, 2
   33 EVNPTS = EVNPTS + CRSECT(I,J)
      DO 34 M = 3, KKK, 2
   34 ODDPTS = ODDPTS + CRSECT(M ,J)
   32 AREASN(J) = (DELTAW/3.)*(CRSECT(1,J) + 4.*EVNPTS + 2.*ODDPTS +
     1CRSECT(NPW,J))
C
C     3RD, WEDDLES RULE
C
      DO 38 J = 1, NPL
      SUMPTS = 0.0
      DO 37 I = 2, NPW, 6
   37 SUMPTS = SUMPTS + 1.*CRSECT(I-1,J) + 5.*CRSECT(I,J)
     1+ 1.*CRSECT(I+1,J) + 6.*CRSECT(I+2,J) + 1.*CRSECT(I+3,J) +
     25.*CRSECT(I+4,J) + 1.*CRSECT(I+5,J)
   38 AREAWD(J) = .3*SUMPTS*DELTAW
      DO 25 I = 1, NPL
      DIFF1(I) = AREASN(I) - AREATR(I)
      DIFF2(I) = AREAWD(I) - AREATR(I)
   25 DIFF3(I) = AREAWD(I) - AREASN(I)
      WRITE (6,35)
   35 FORMAT(15X, 6HAREATR, 10X, 6HAREASN, 10X, 6HAREAWD, 10X, 5HDIFF1,
     15X, 5HDIFF2, 5X, 5HDIFF3//)
      WRITE (6,36) (I, AREATR(I), AREASN(I), AREAWD(I),DIFF1(I),
     1DIFF2(I), DIFF3(I), I = 1, NPL)
   36 FORMAT(10X, I3, F10.4, 6X, F10.4, 6X, F10.4, 5X, F7.4, 3X, F7.4,
     13X, F7.4)
      WRITE (6,39)
   39 FORMAT (3X)
C
C     COMPUTATION OF VOLUME USING TRAPEZOIDAL, SIMPSONS AND WEDDLES
C     RULES.
C
C     1ST, AREATR AND LENGTH USING TRAPEZOIDAL RULE
C
      N = NPL - 1
      SUMA = 0.0
      DO 40 I = 2, N
   40 SUMA = SUMA + AREATR(I)
      VCTRTR = (DELTAL/2.0) * (AREATR(1) + 2.*SUMA + AREATR(NPL))
      PETRTR = 100.0*(VCTRTR - VACT)/VACT
C
C     2ND, AREATR AND LENGTH USING SIMPSONS RULE.
C
      N = NPL - 1
      NN = N - 1
      ODAREA = 0.0
      EVAREA = 0.0
      DO 50 I = 2, N, 2
   50 EVAREA = EVAREA + AREATR(I)
      DO 51 I = 3, NN, 2
   51 ODAREA = ODAREA + AREATR(I)
      VCTRSN = (DELTAL/3.)*(AREATR(1) + 4.*EVAREA + 2.*ODAREA +
     1AREATR(NPL))
      PETRSN = 100.0*(VCTRSN - VACT)/VACT
C     3RD, AREASN AND LENGTH USING TRAPEZOIDAL RULE
```

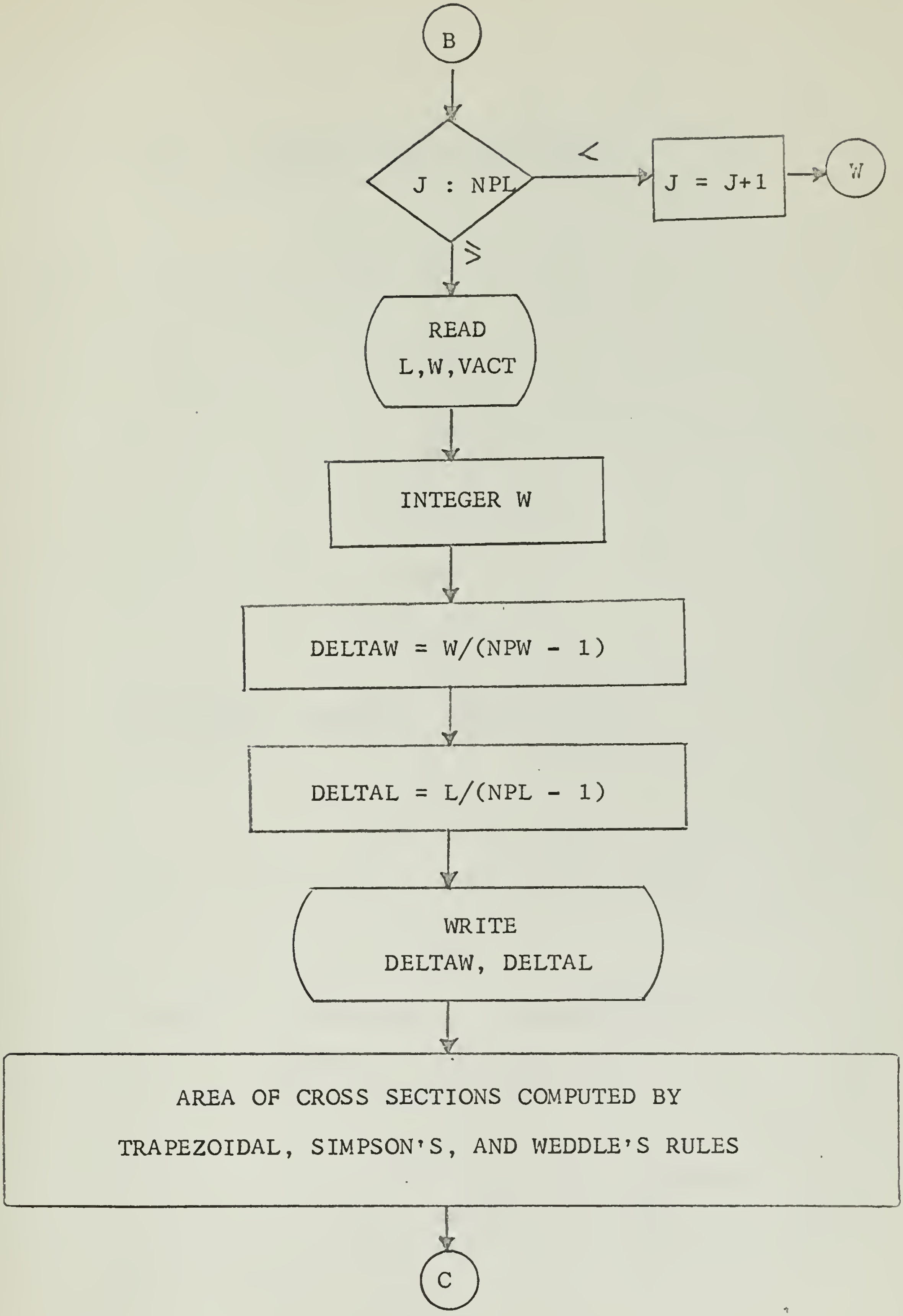
```
      N = NPL - 1
      SUMA = 0.0
      DO 60 I = 2, N
   60 SUMA = SUMA + AREASN(I)
      VCSNTR = (DELTAL/2.)*(AREASN(1) + 2.*SUMA + AREASN(NPL))
      PESNTR = 100.0*(VCSNTR - VACT)/VACT
C
C     4TH, AREASN AND LENGTH USING SIMPSONS RULE
C
      N = NPL - 1
      NN = N - 1
      ODAREA = 0.0
      EVAREA = 0.0
      DO 70 I = 2, N, 2
   70 EVAREA = EVAREA + AREASN(I)
      DO 71 I = 3, NN, 2
   71 ODAREA = ODAREA + AREASN(I)
      VCSNSN = (DELTAL/3.)*(AREASN(1) + 4.*EVAREA + 2.*ODAREA +
     1AREASN(NPL))
      PESNSN = 100.0*(VCSNSN - VACT)/VACT
C
C     5TH, AREATR AND LENGTH USING WEDDLES RULE
C
      SUMA = 0.0
      DO 75 I = 2, NPL, 6
   75 SUMA = SUMA +1.*AREATR(I-1) + 5.*AREATR(I) +1.*AREATR(I+1) +
     16.*AREATR(I+2) + 1.*AREATR(I+3) + 5.*AREATR(I+4) + 1.*AREATR(I+5)
      VCTRWD = .3*SUMA*DELTAL
      PETRWD = 100.0*(VCTRWD - VACT)/VACT
C
C     6TH, AREASN AND LENGTH USING WEDDLES RULE
C
      SUMA = 0.0
      DO 76 I = 2, NPL, 6
   76 SUMA = SUMA + 1.*AREASN(I-1) + 5.*AREASN(I) + 1.*AREASN(I+1) +
     16.*AREASN(I+2) + 1.*AREASN(I+3) + 5.*AREASN(I+4) + 1.*AREASN(I+5)
      VCSNWD = .3*SUMA*DELTAL
      PESNWD = 100.0*(VCSNWD - VACT)/VACT
C
C     7TH, AREAWD AND LENGTH USING TRAPEZOIDAL RULE
C
      N = NPL - 1
      SUMA = 0.0
      DO 77 I = 2, N
   77 SUMA = SUMA + AREAWD(I)
      VCWDTR = (DELTAL/2.)*(AREAWD(1) + 2.*SUMA + AREAWD(NPL))
      PEWDTR = 100.0*(VCWDTR -VACT)/VACT
C
C     8TH, AREAWD AND LENGTH USING SIMPSONS RULE
C
      N = NPL - 1
      NN = N- 1
      ODAREA = 0.0
      EVAREA = 0.0
      DO 78 I = 2, N, 2
   78 EVAREA = EVAREA + AREAWD(I)
      DO 79 I = 3, NN, 2
```

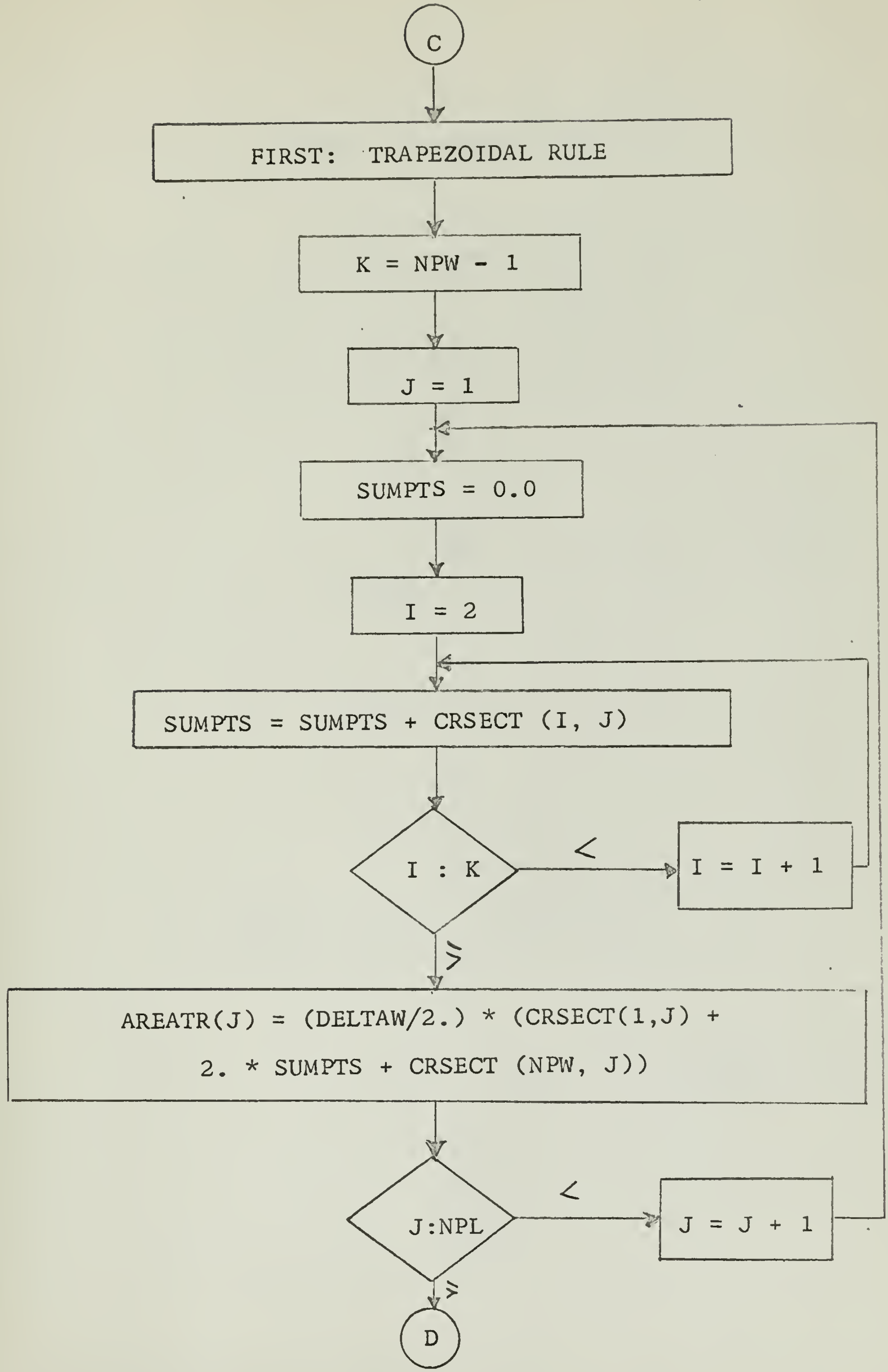
```
79 ODAREA = ODAREA + AREAWD(I)
   VCWDSN = (DELTAL/3.)*(AREAWD(1) + 4.*EVAREA + 2.*ODAREA
  1+ AREAWD(NPL))
   PEWDSN = 100.0*(VCWDSN - VACT)/VACT
C
C     9TH, AREAWD AND LENGTH USING WEDDLES RULE
C
   SUMA = 0.0
   DO 80 I = 2, NPL, 6
80 SUMA = SUMA + 1.*AREAWD(I-1) + 5.*AREAWD(I) + 1.*AREAWD(I+1) +
  16.*AREAWD(I+2) +1.*AREAWD(I+3) + 5.*AREAWD(I+4) + 1.*AREAWD(I+5)
   VCWDWD = .3*SUMA*DELTAL
   PEWDWD = 100.0*(VCWDWD - VACT)/VACT
C
   WRITE(6,41)    VACT
41 FORMAT (50X, 10HVACTUAL = , F10.2//)
   WRITE (6,42)
42 FORMAT(13X, 6HVCTRTR, 6X, 6HVCTRSN, 6X, 6HVCSNTR, 6X, 6HVCSNSN, 6X
  1, 6HVCTRWD, 6X, 6HVCSNWD, 6X. 6HVCWDTR, 6X, 6HVCWDSN, 6X, 6HVCWDWD
  2, 6X//)
   WRITE(6,52) VCTRTR, VCTRSN, VCSNTR, VCSNSN, VCTRWD, VCSNWD,
  1VCWDTR, VCWDSN, VCWDWD
52 FORMAT(10X, F10.2, 2X, F10.2, 2X, F10.2, 2X, F10.2, 2X, F10.2, 2X,
  1F10.2, 2X, F10.2, 2X, F10.2,  2X, F10.2, 2X//)
   WRITE (6,62)
62 FORMAT (13X, 6HPETRTR, 6X, 6HPETRSN, 6X, 6HPESNTR, 6X,
  16HPESNSN, 6X, 6HPETRWD, 6X, 6HPESNWD, 6X, 6HPEWDTR, 6X, 6HPEWDSN,
  26X, 6HPEWDWD, 6X//)
   WRITE(6,72) PETRTR, PETRSN, PESNTR, PESNSN, PETRWD, PESNWD,
  1PEWDTR, PEWDSN, PEWDWD
72 FORMAT(13X, F6.2, 6X, F6.2, 6X, F6.2, 6X, F6.2, 6X, F6.2, 6X,
  1F6.2, 6X, F6.2, 6X, F6.2, 6X, F6.2//)
C
   CALL EXIT
   END
```

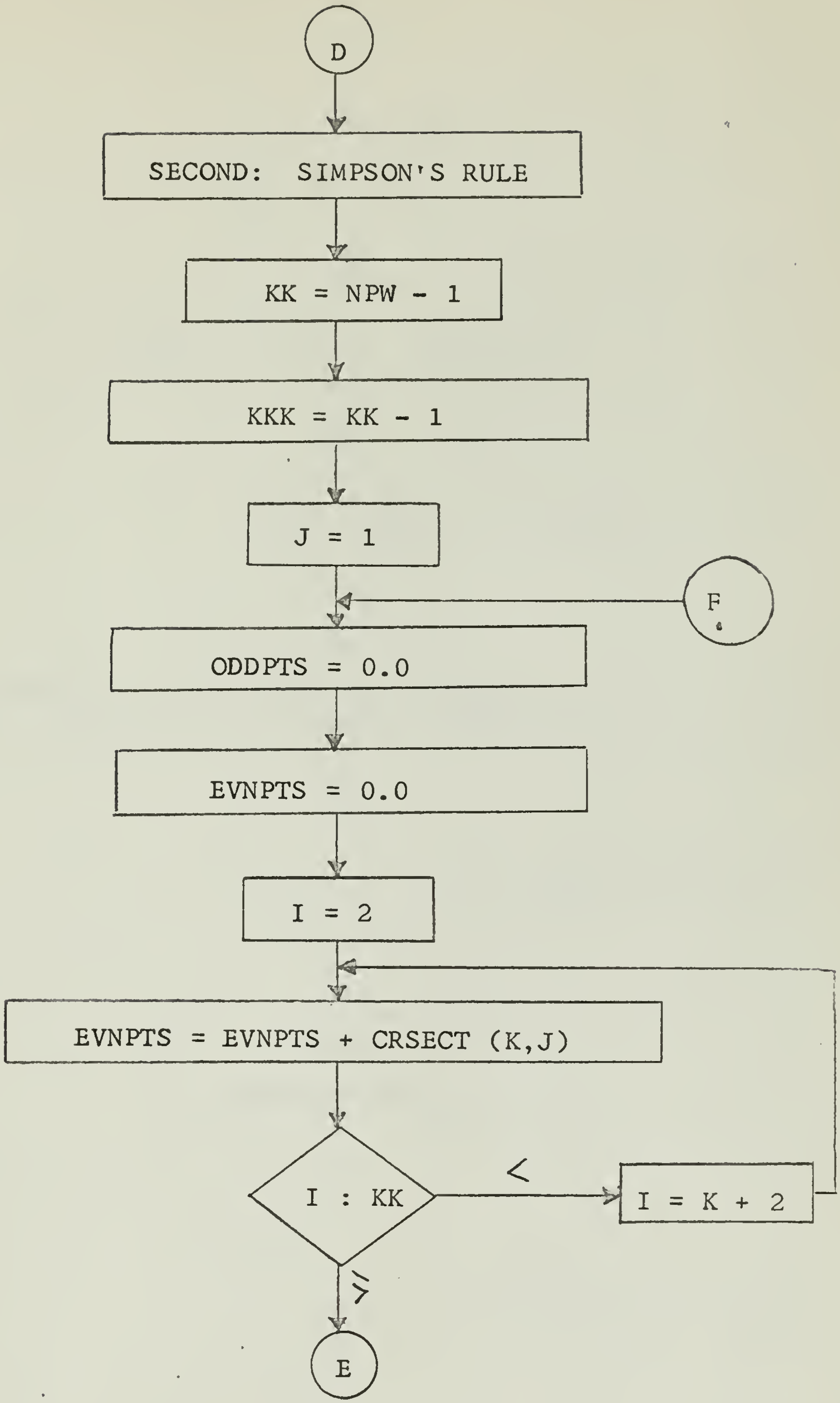
FLOW DIAGRAM FOR COMPUTER PROGRAM TO DETERMINE THE APPROXIMATE VOLUME OF A STOCKPILE BY DOUBLE NUMERICAL INTEGRATION UTILIZING THE TRAPEZOIDAL, SIMPSON'S, AND WEDDLE'S QUADRATURE RULES.



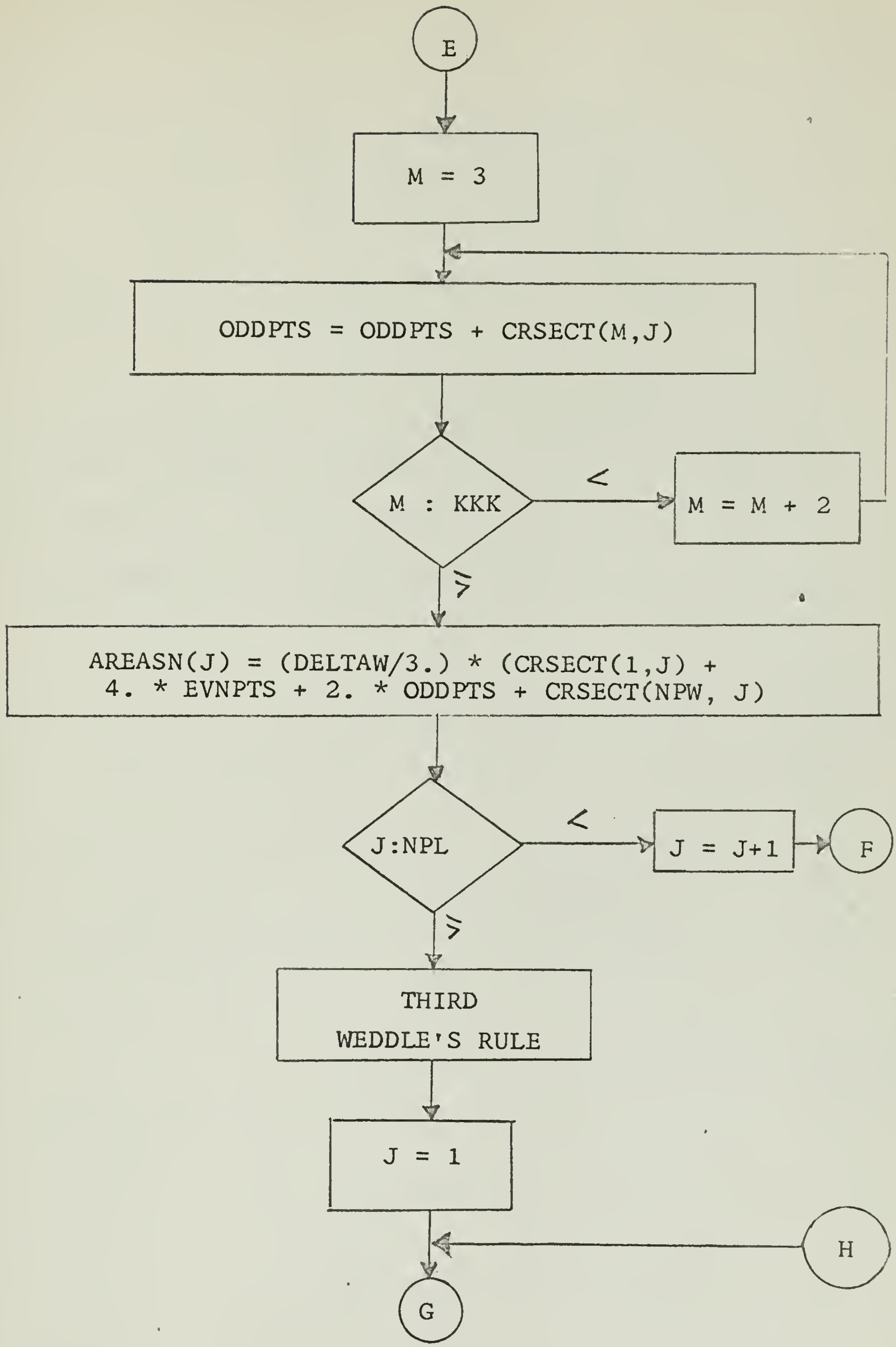

A
NP2 = NPW - 1
READ
((CRSECT (I,J), I = 2, NP2), J = 1, NPL)
CONVERTING DATA VALUE INTO INCHES
J = 1
W
I = 1
CRSECT (I,J) = CRSECT (I,J) * 12.0
I = I + 1
<
I : NPW
≥
B

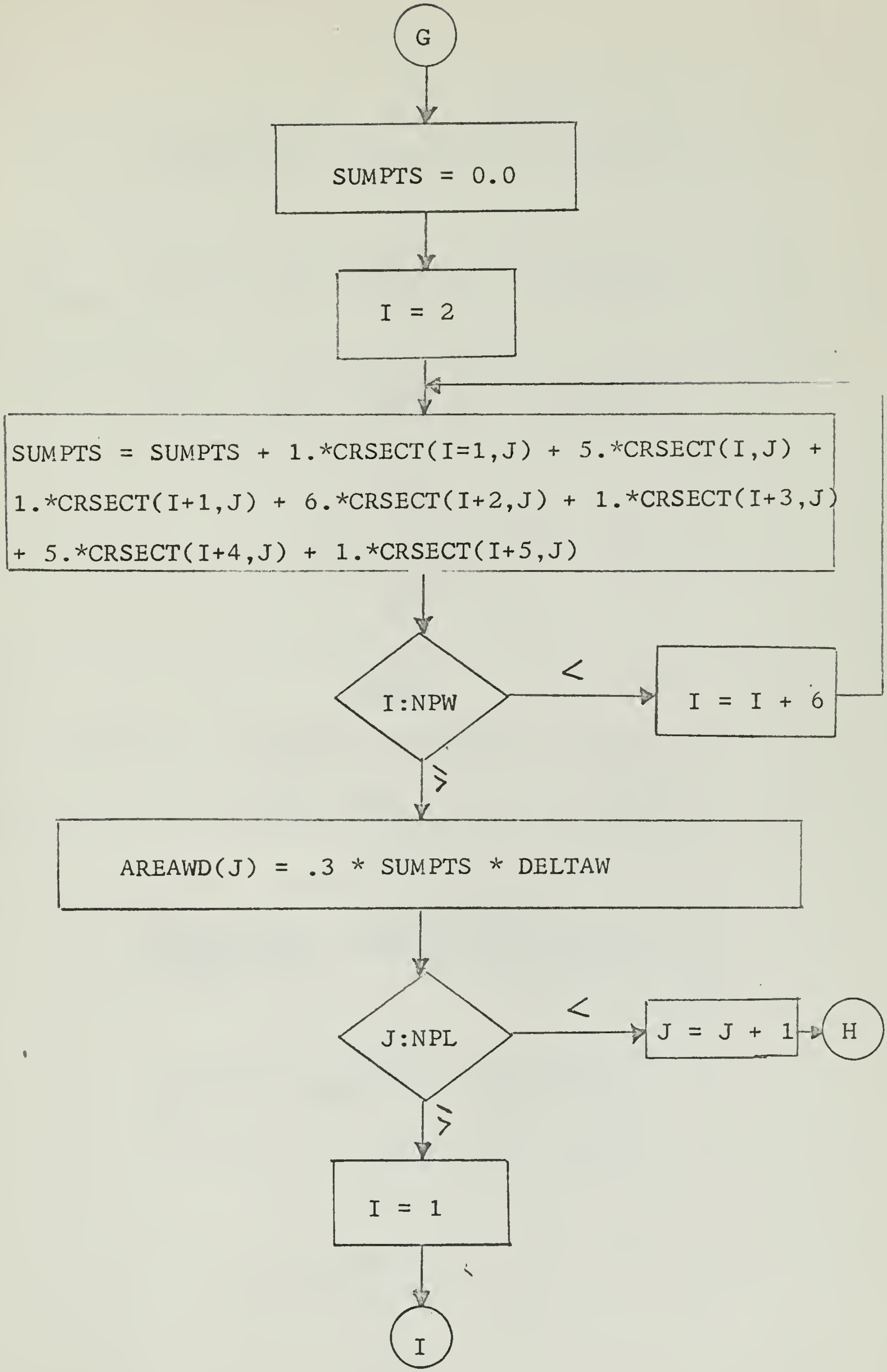

B
J : NPL
<
J = J+1
W
≥
READ
L,W,VACT
INTEGER W
DELTAW = W/(NPW - 1)
DELTAL = L/(NPL - 1)
WRITE
DELTAW, DELTAL
AREA OF CROSS SECTIONS COMPUTED BY
TRAPEZOIDAL, SIMPSON'S, AND WEDDLE'S RULES
C

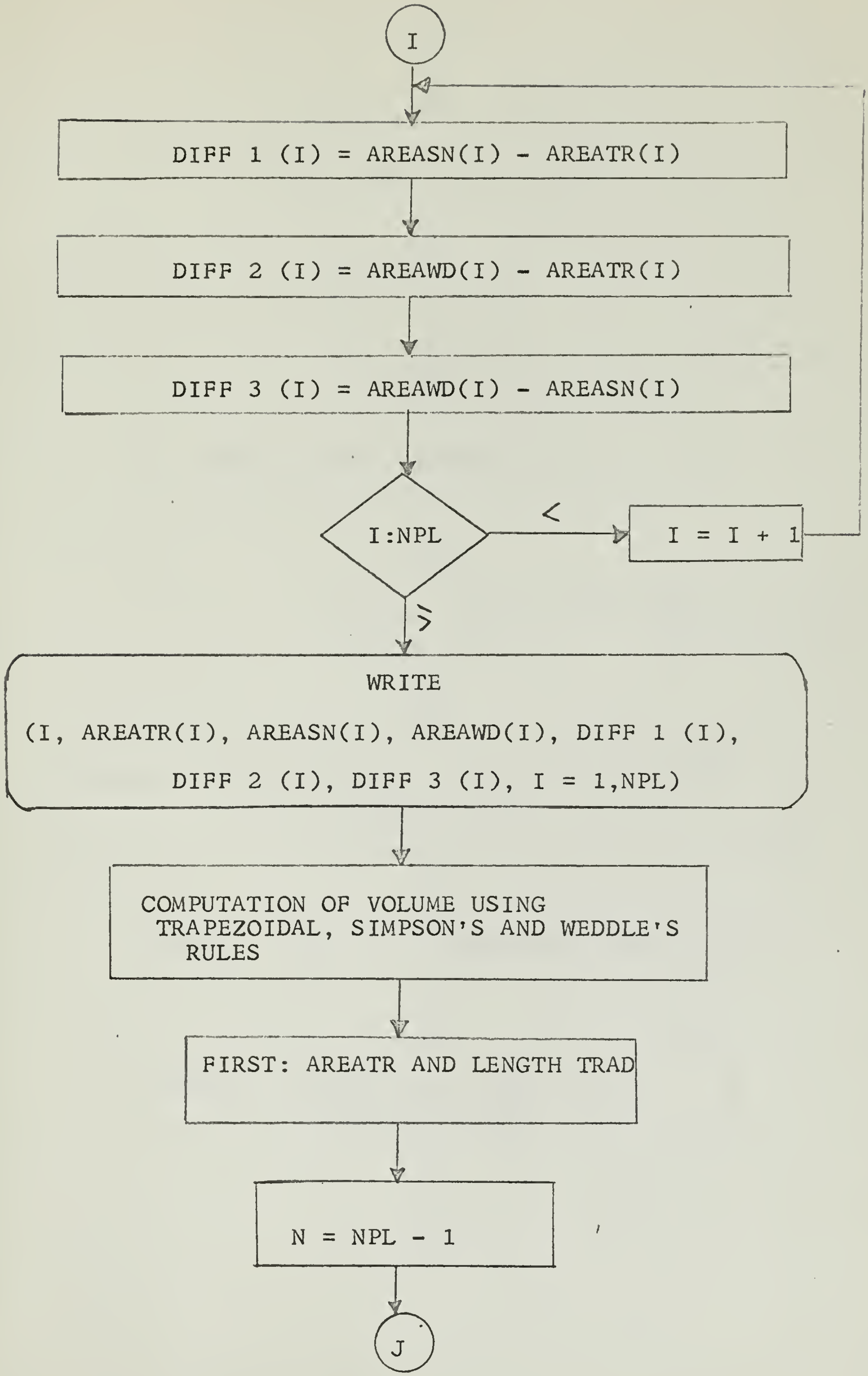

C
FIRST: TRAPEZOIDAL RULE
K = NPW - 1
J = 1
SUMPTS = 0.0
I = 2
SUMPTS = SUMPTS + CRSECT (I, J)
I : K
<
I = I + 1
≥
AREATR(J) = (DELTAW/2.) * (CRSECT(1,J) +
2. * SUMPTS + CRSECT (NPW, J))
J:NPL
<
J = J + 1
≥
D

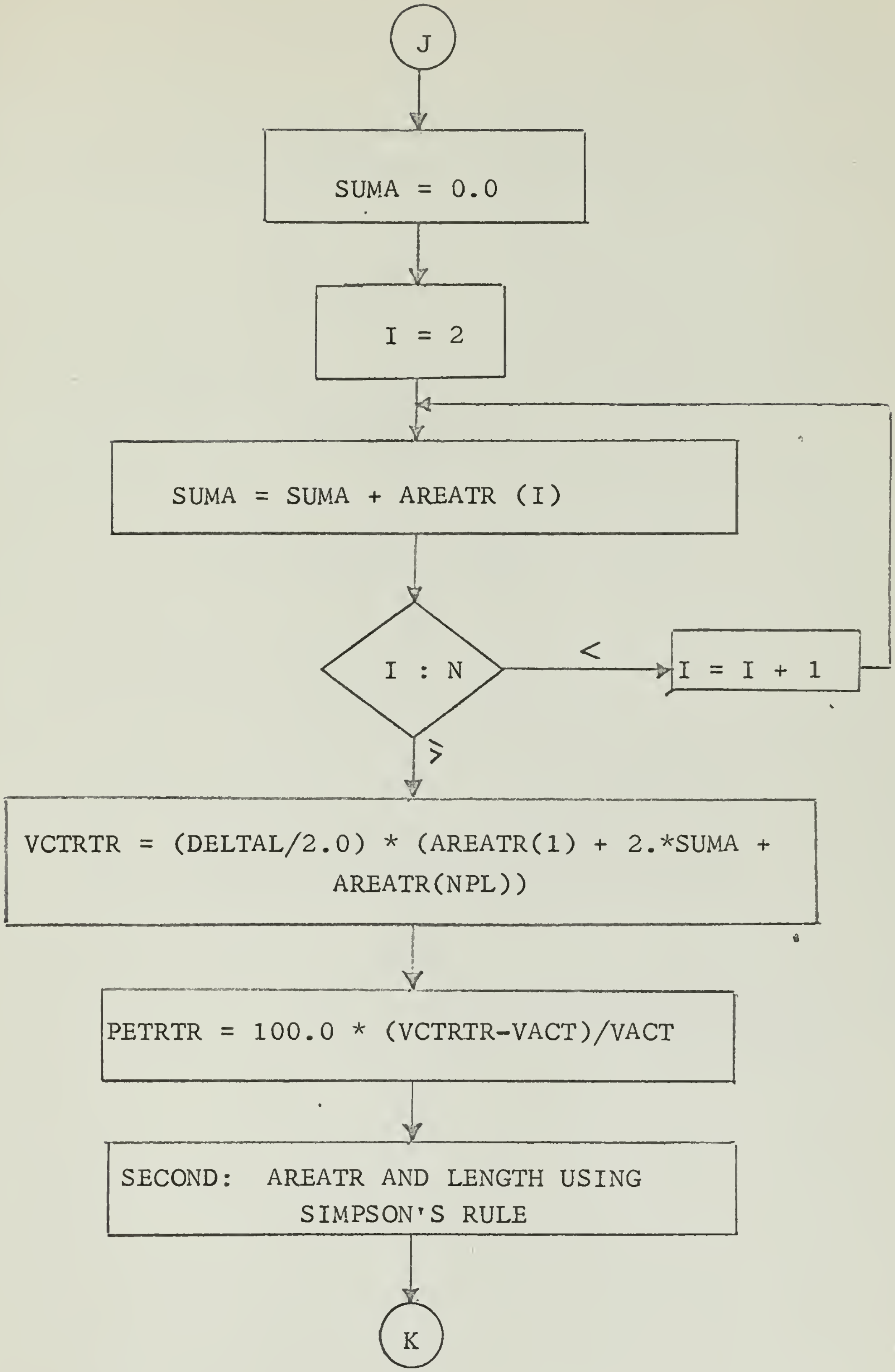


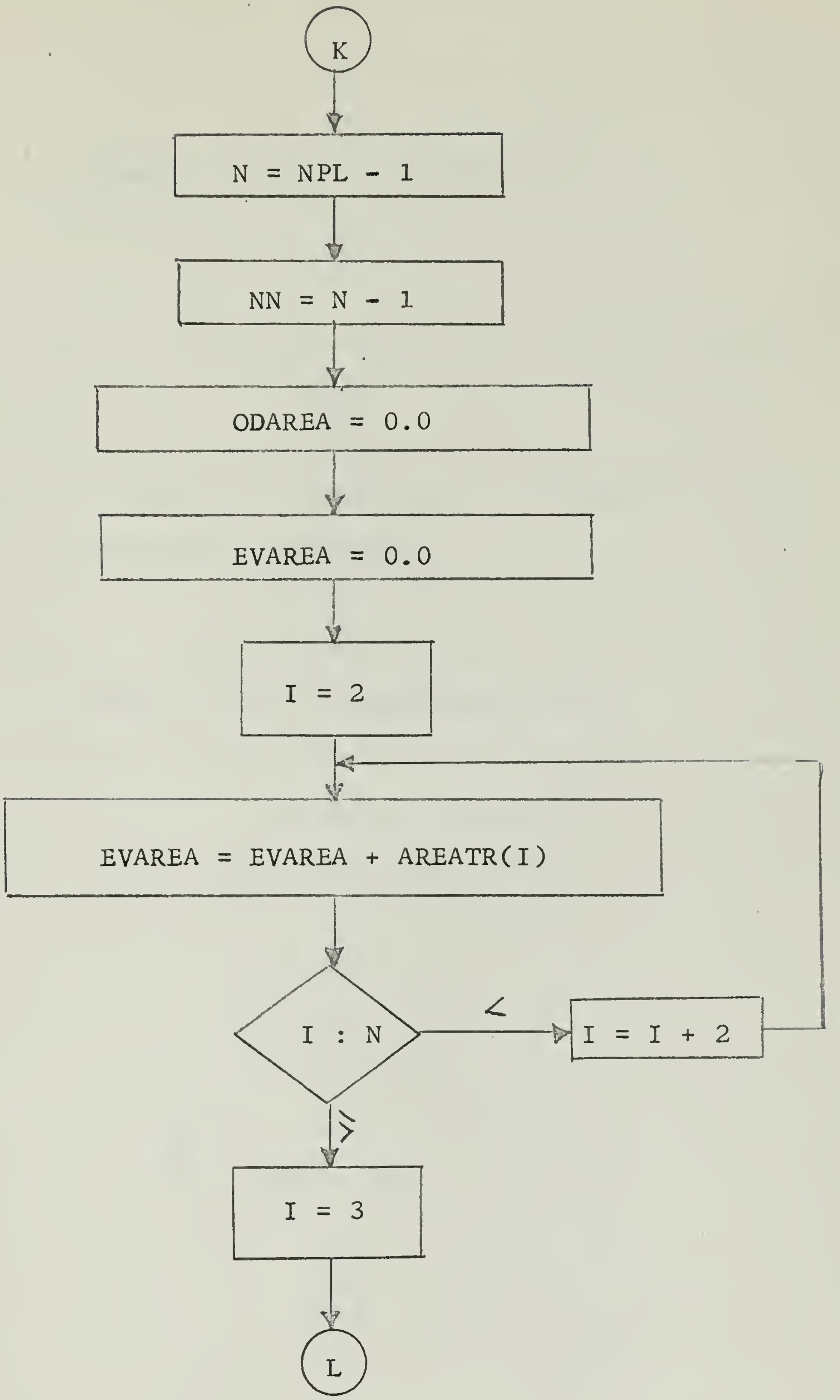
D
SECOND: SIMPSON'S RULE
KK = NPW - 1
KKK = KK - 1
J = 1
F
ODDPTS = 0.0
EVNPTS = 0.0
I = 2
EVNPTS = EVNPTS + CRSECT (K,J)
I : KK
<
I = K + 2
≥
E

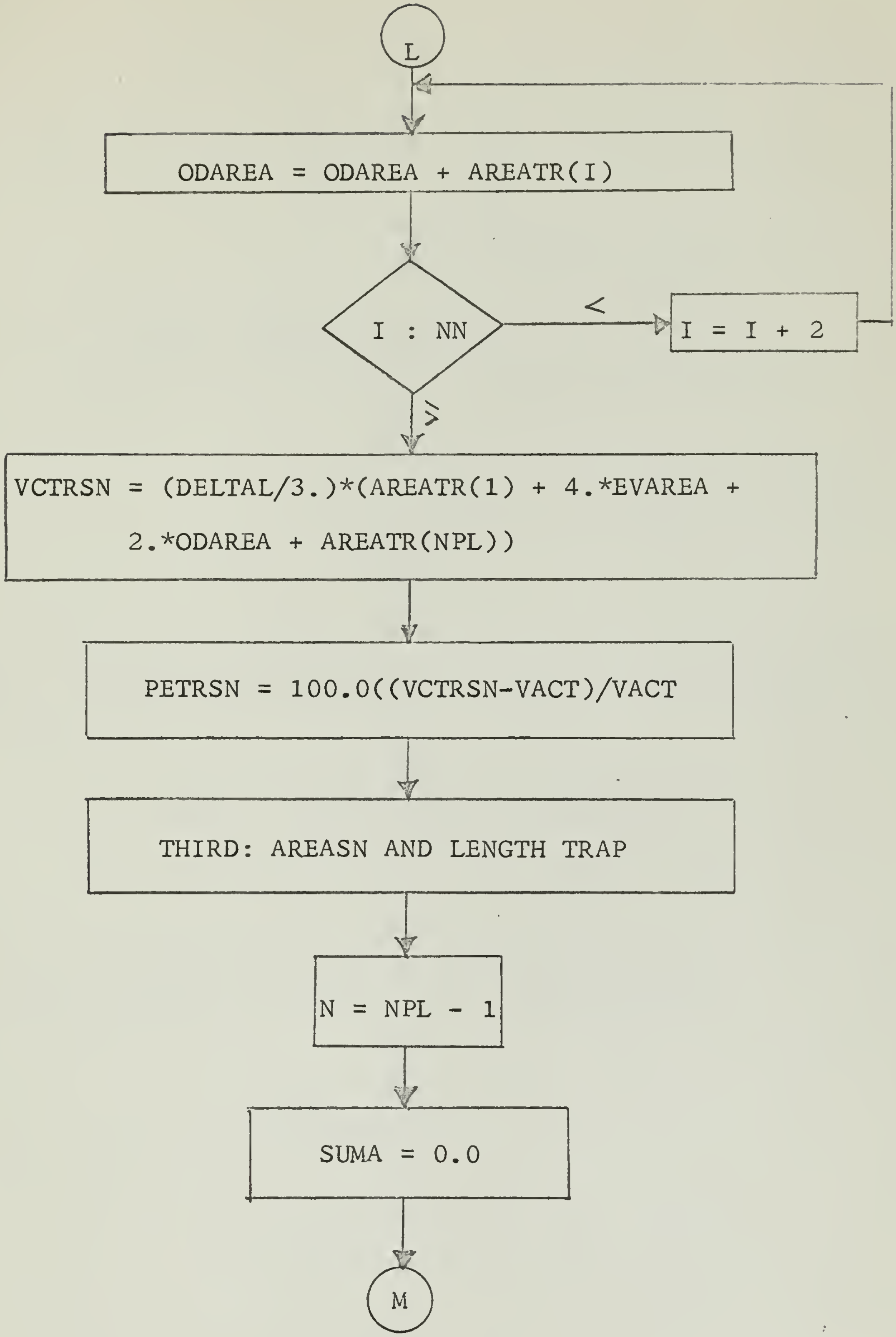

E
M = 3
ODDPTS = ODDPTS + CRSECT(M,J)
M : KKK
<
M = M + 2
≥
AREASN(J) = (DELTAW/3.) * (CRSECT(1,J) + 4. * EVNPTS + 2. * ODDPTS + CRSECT(NPW, J)
J:NPL
<
J = J+1
F
≥
THIRD WEDDLE'S RULE
J = 1
H
G

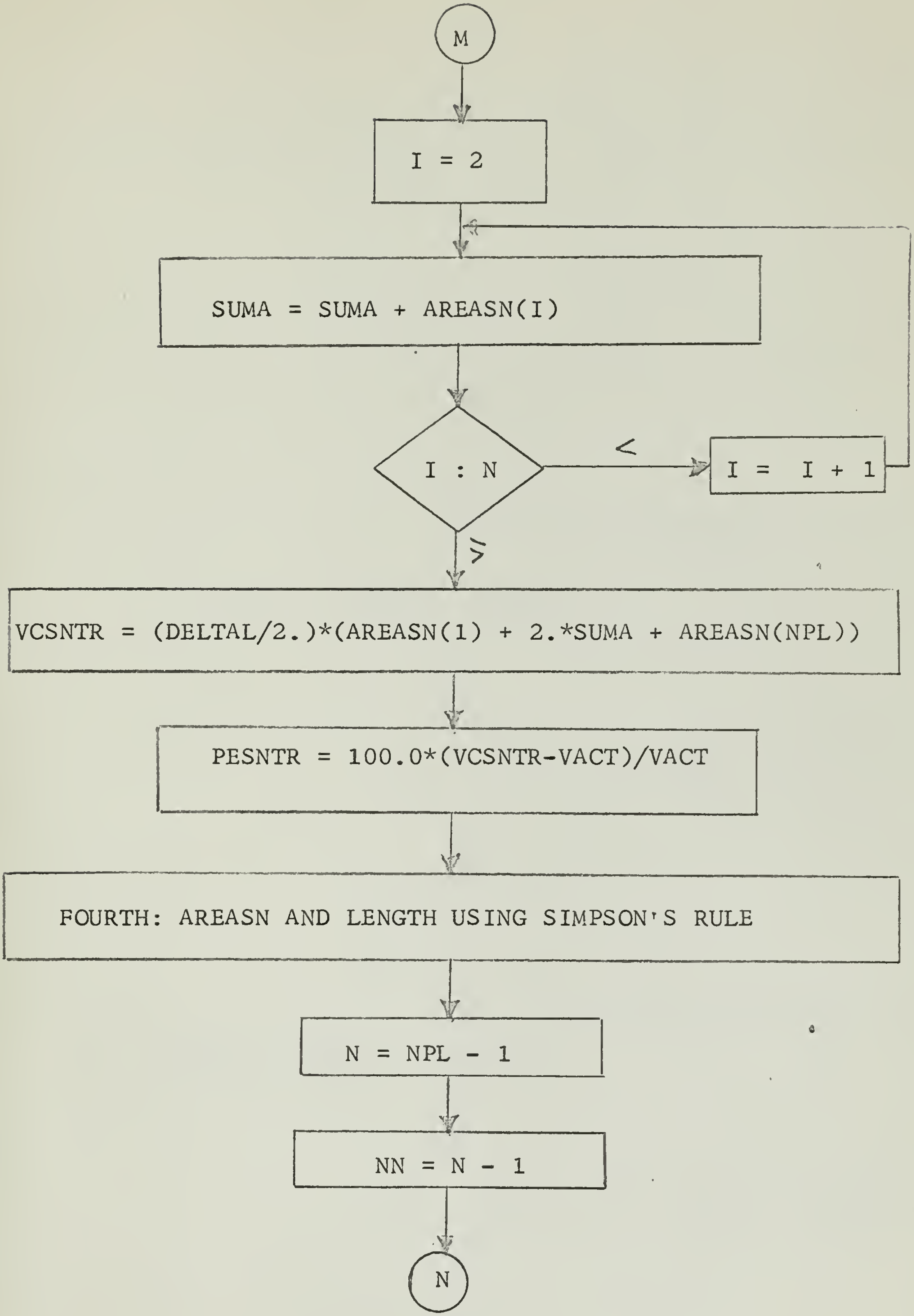

G
SUMPTS = 0.0
I = 2
SUMPTS = SUMPTS + 1.*CRSECT(I=1,J) + 5.*CRSECT(I,J) +
1.*CRSECT(I+1,J) + 6.*CRSECT(I+2,J) + 1.*CRSECT(I+3,J)
+ 5.*CRSECT(I+4,J) + 1.*CRSECT(I+5,J)
I:NPW
<
I = I + 6
≥
AREAWD(J) = .3 * SUMPTS * DELTAW
J:NPL
<
J = J + 1
H
≥
I = 1
I

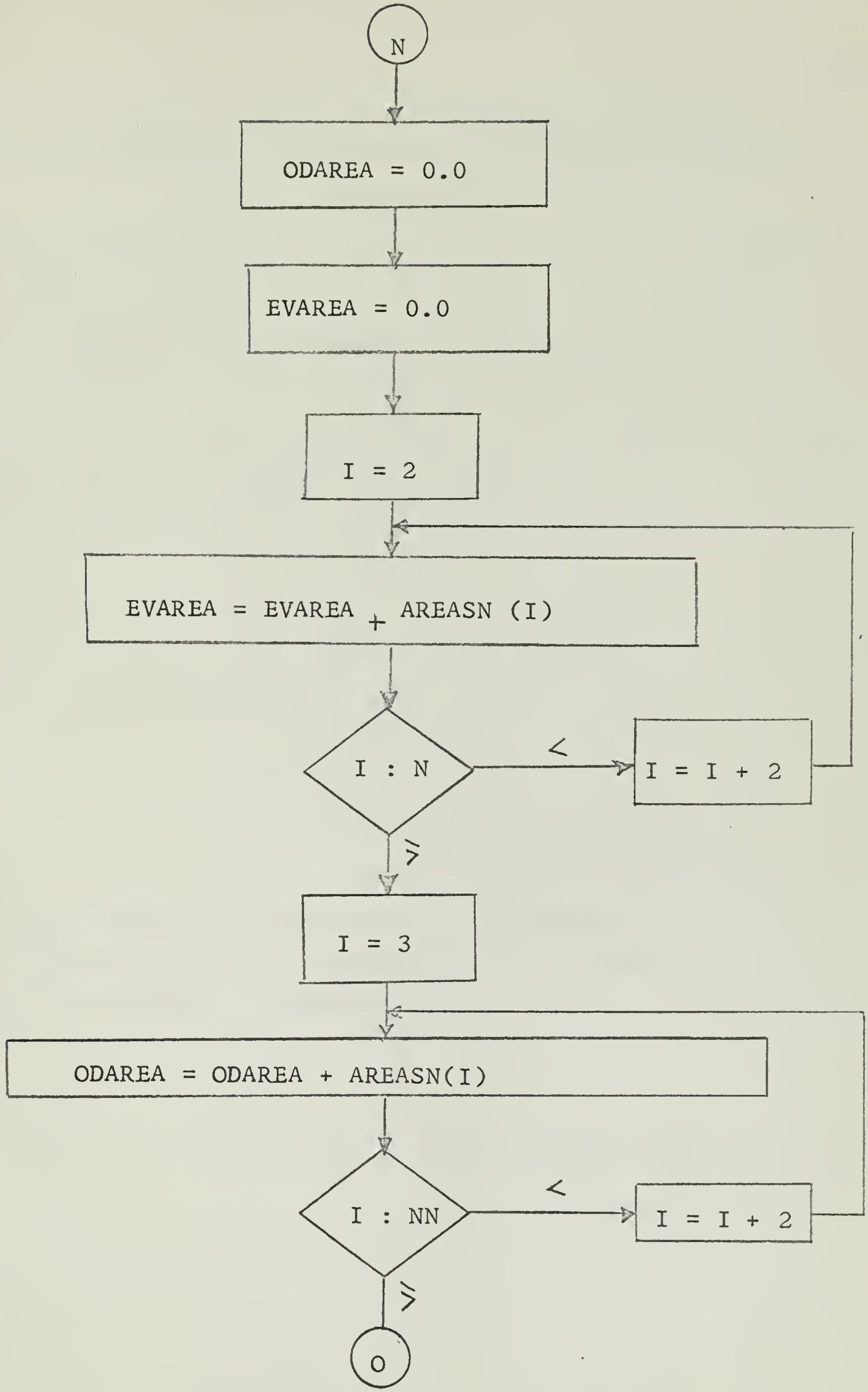

I
DIFF 1 (I) = AREASN(I) - AREATR(I)
DIFF 2 (I) = AREAWD(I) - AREATR(I)
DIFF 3 (I) = AREAWD(I) - AREASN(I)
I:NPL
<
I = I + 1
≥
WRITE
(I, AREATR(I), AREASN(I), AREAWD(I), DIFF 1 (I), DIFF 2 (I), DIFF 3 (I), I = 1,NPL)
COMPUTATION OF VOLUME USING TRAPEZOIDAL, SIMPSON'S AND WEDDLE'S RULES
FIRST: AREATR AND LENGTH TRAD
N = NPL - 1
J

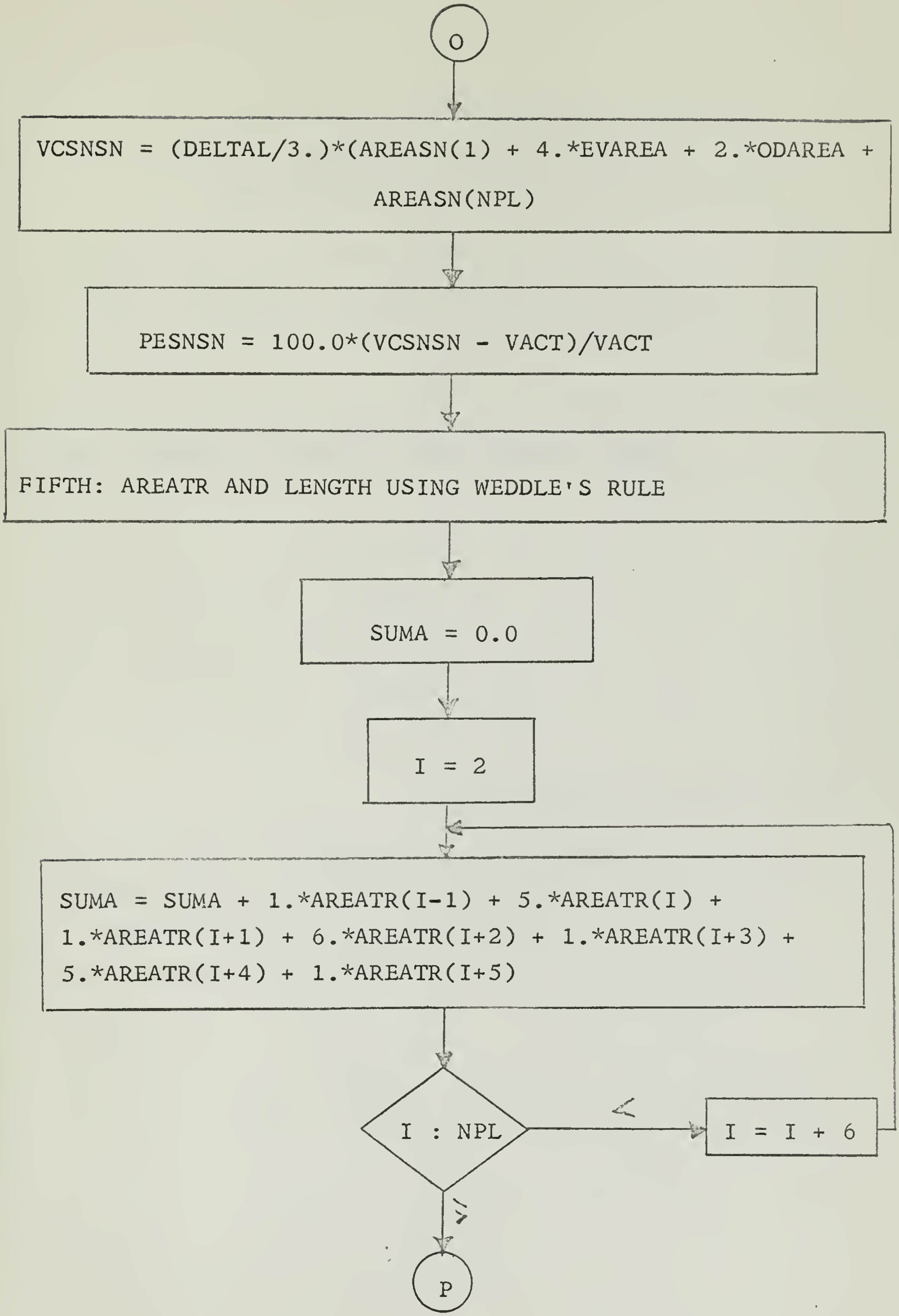

J
SUMA = 0.0
I = 2
SUMA = SUMA + AREATR (I)
I : N
<
I = I + 1
≥
VCTRTR = (DELTAL/2.0) * (AREATR(1) + 2.*SUMA + AREATR(NPL))
PETRTR = 100.0 * (VCTRTR-VACT)/VACT
SECOND: AREATR AND LENGTH USING SIMPSON'S RULE
K

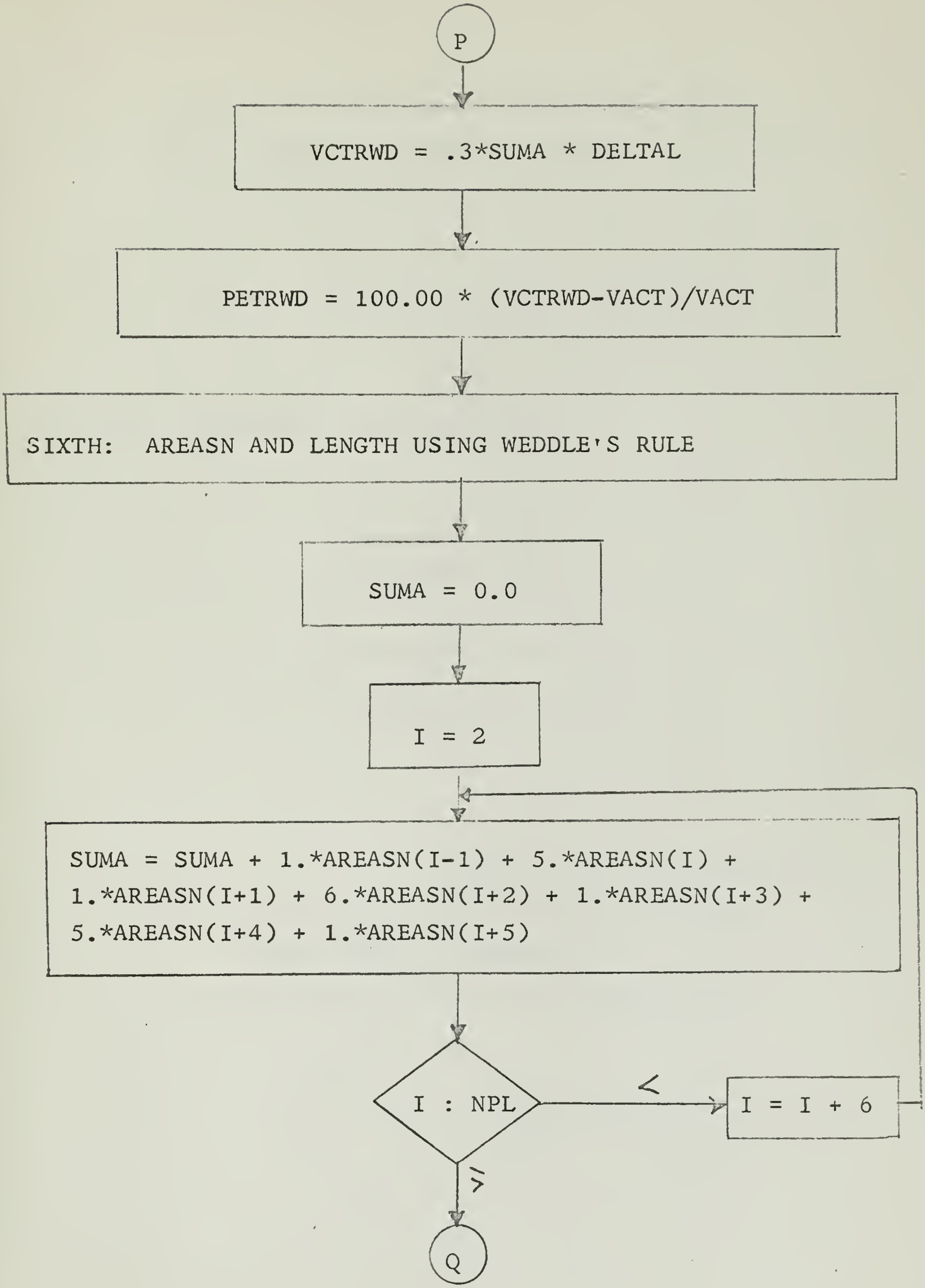

K
N = NPL - 1
NN = N - 1
ODAREA = 0.0
EVAREA = 0.0
I = 2
EVAREA = EVAREA + AREATR(I)
I : N
<
I = I + 2
≥
I = 3
L

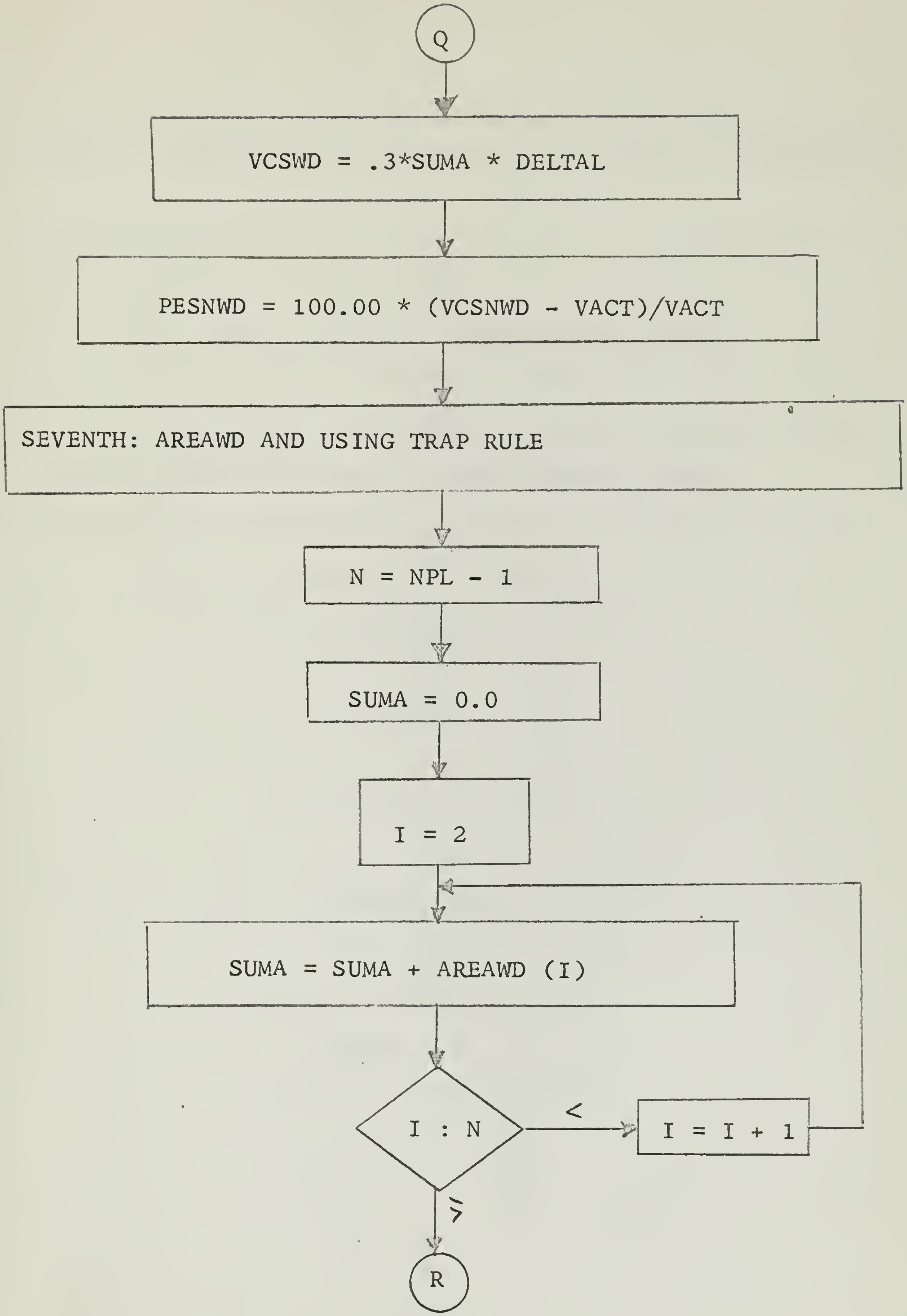

L
ODAREA = ODAREA + AREATR(I)
I : NN
<
I = I + 2
≥
VCTRSN = (DELTAL/3.)*(AREATR(1) + 4.*EVAREA +
2.*ODAREA + AREATR(NPL))
PETRSN = 100.0((VCTRSN-VACT)/VACT
THIRD: AREASN AND LENGTH TRAP
N = NPL - 1
SUMA = 0.0
M

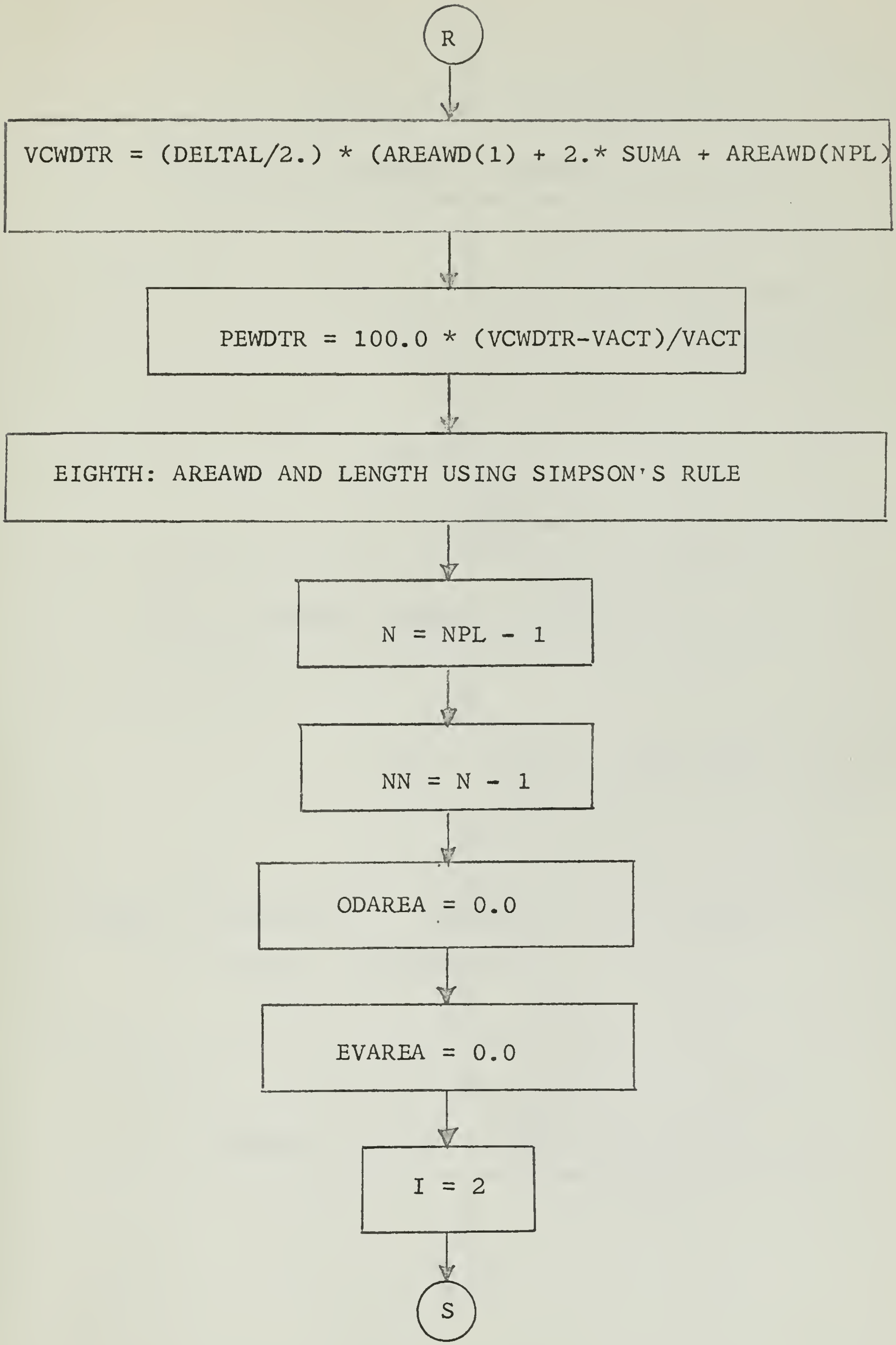

M
I = 2
SUMA = SUMA + AREASN(I)
I : N
<
I = I + 1
≥
VCSNTR = (DELTAL/2.)*(AREASN(1) + 2.*SUMA + AREASN(NPL))
PESNTR = 100.0*(VCSNTR-VACT)/VACT
FOURTH: AREASN AND LENGTH USING SIMPSON'S RULE
N = NPL - 1
NN = N - 1
N

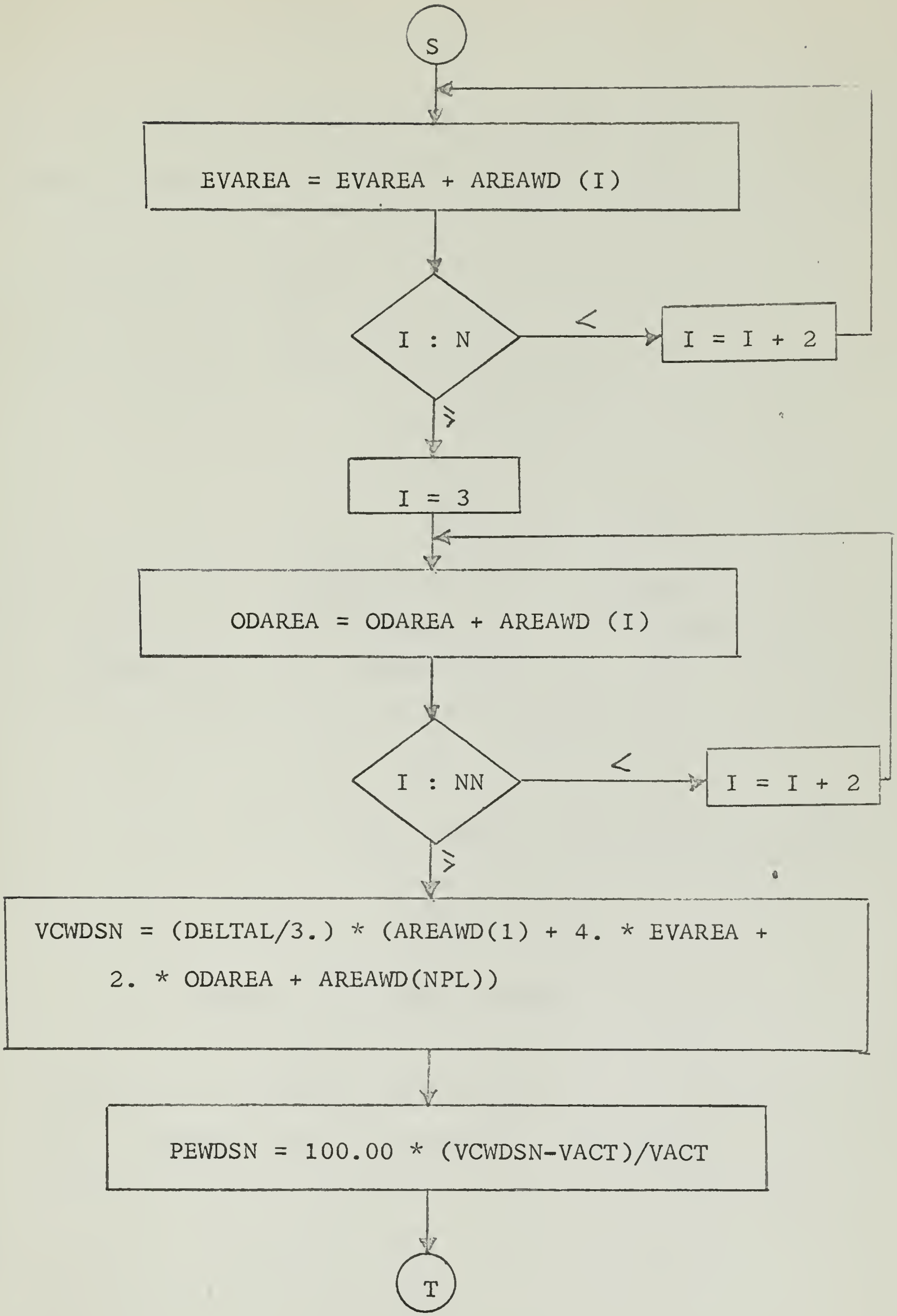

N
ODAREA = 0.0
EVAREA = 0.0
I = 2
EVAREA = EVAREA + AREASN (I)
I : N
<
I = I + 2
≥
I = 3
ODAREA = ODAREA + AREASN(I)
I : NN
<
I = I + 2
≥
O

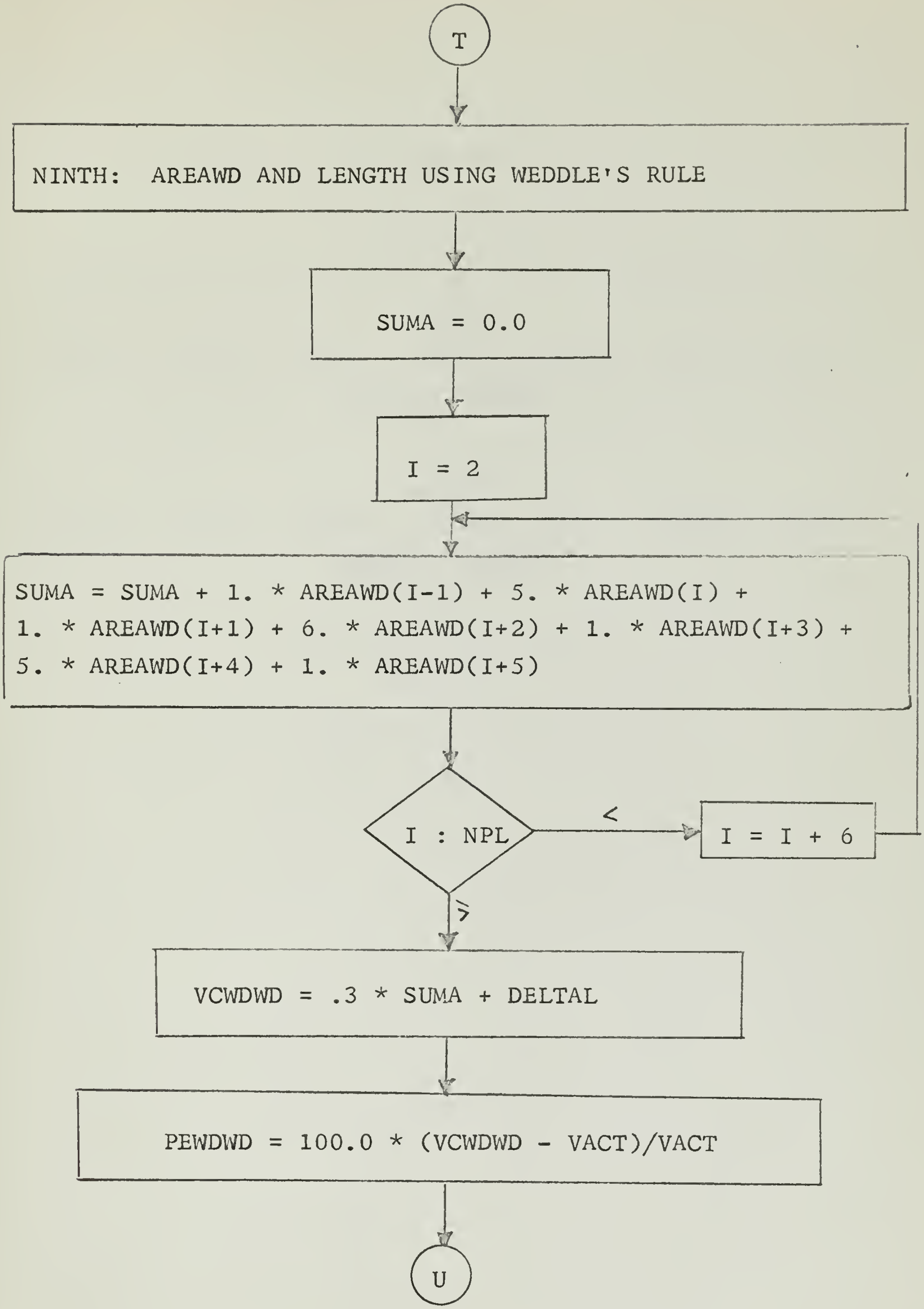

O
VCSNSN = (DELTAL/3.)*(AREASN(1) + 4.*EVAREA + 2.*ODAREA + AREASN(NPL)
PESNSN = 100.0*(VCSNSN - VACT)/VACT
FIFTH: AREATR AND LENGTH USING WEDDLE'S RULE
SUMA = 0.0
I = 2
SUMA = SUMA + 1.*AREATR(I-1) + 5.*AREATR(I) + 1.*AREATR(I+1) + 6.*AREATR(I+2) + 1.*AREATR(I+3) + 5.*AREATR(I+4) + 1.*AREATR(I+5)
I : NPL
<
I = I + 6
>
P

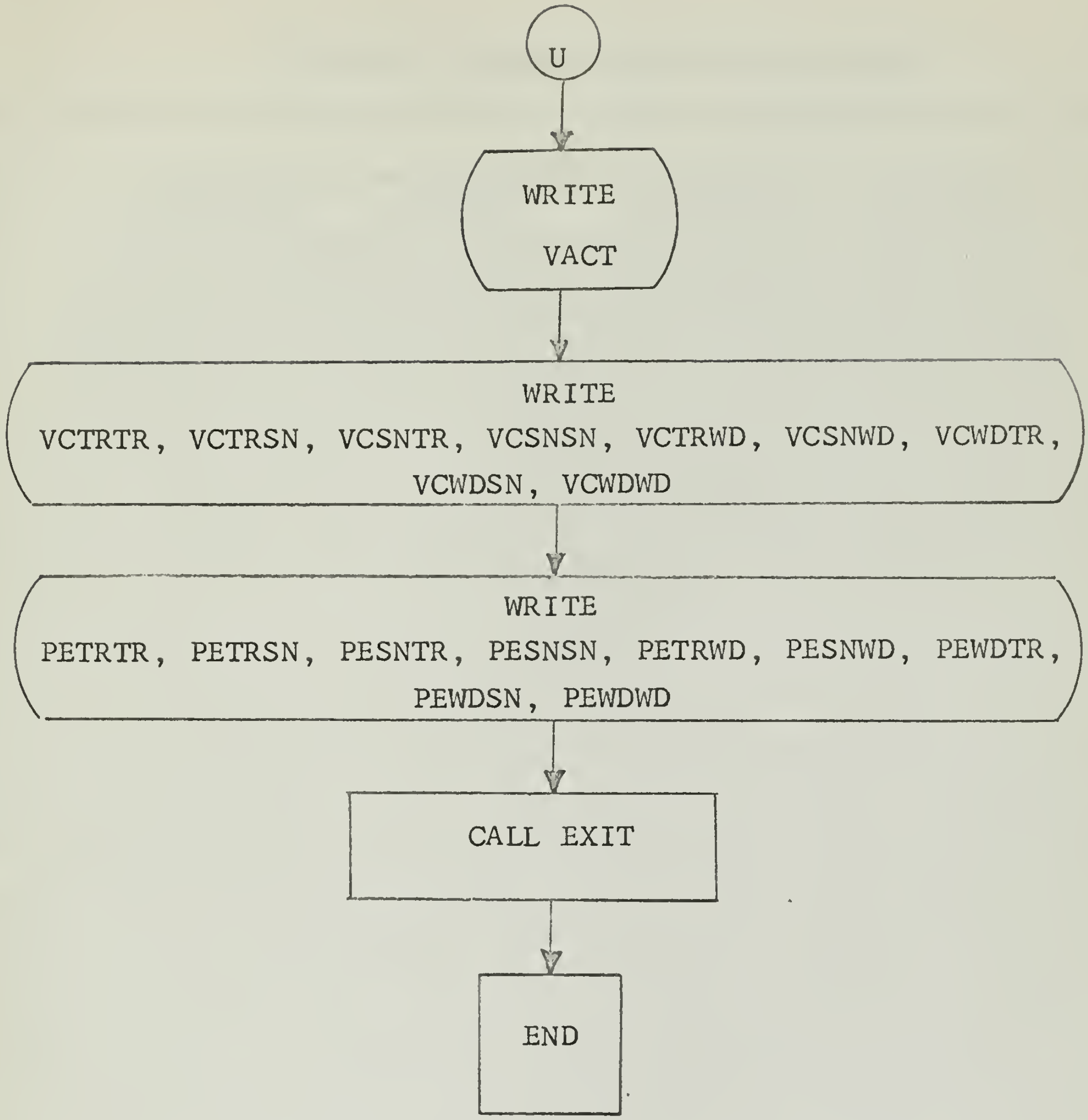

P
VCTRWD = .3*SUMA * DELTAL
PETRWD = 100.00 * (VCTRWD-VACT)/VACT
SIXTH: AREASN AND LENGTH USING WEDDLE'S RULE
SUMA = 0.0
I = 2
SUMA = SUMA + 1.*AREASN(I-1) + 5.*AREASN(I) + 1.*AREASN(I+1) + 6.*AREASN(I+2) + 1.*AREASN(I+3) + 5.*AREASN(I+4) + 1.*AREASN(I+5)
I : NPL
<
I = I + 6
≥
Q


Q
VCSWD = .3*SUMA * DELTAL
PESNWD = 100.00 * (VCSNWD - VACT)/VACT
SEVENTH: AREAWD AND USING TRAP RULE
N = NPL - 1
SUMA = 0.0
I = 2
SUMA = SUMA + AREAWD (I)
I : N
<
I = I + 1
≥
R


R
VCWDTR = (DELTAL/2.) * (AREAWD(1) + 2.* SUMA + AREAWD(NPL)
PEWDTR = 100.0 * (VCWDTR-VACT)/VACT
EIGHTH: AREAWD AND LENGTH USING SIMPSON'S RULE
N = NPL - 1
NN = N - 1
ODAREA = 0.0
EVAREA = 0.0
I = 2
S


S
EVAREA = EVAREA + AREAWD (I)
I : N
<
I = I + 2
≥
I = 3
ODAREA = ODAREA + AREAWD (I)
I : NN
<
I = I + 2
≥
VCWDSN = (DELTAL/3.) * (AREAWD(1) + 4. * EVAREA +
2. * ODAREA + AREAWD(NPL))
PEWDSN = 100.00 * (VCWDSN-VACT)/VACT
T


T
NINTH: AREAWD AND LENGTH USING WEDDLE'S RULE
SUMA = 0.0
I = 2
SUMA = SUMA + 1. * AREAWD(I-1) + 5. * AREAWD(I) +
1. * AREAWD(I+1) + 6. * AREAWD(I+2) + 1. * AREAWD(I+3) +
5. * AREAWD(I+4) + 1. * AREAWD(I+5)
I : NPL
<
I = I + 6
≥
VCWDWD = .3 * SUMA + DELTAL
PEWDWD = 100.0 * (VCWDWD - VACT)/VACT
U


U
WRITE
VACT
WRITE
VCTRTR, VCTRSN, VCSNTR, VCSNSN, VCTRWD, VCSNWD, VCWDTR, VCWDSN, VCWDWD
WRITE
PETRTR, PETRSN, PESNTR, PESNSN, PETRWD, PESNWD, PEWDTR, PEWDSN, PEWDWD
CALL EXIT
END

Undisturbed Stockpile Depth Measurements

MEASUREMENT LOCATIONS (25)

CROSS SECTIONS (61)

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.001	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.002	0.044	0.060	0.069	0.067	0.066	0.051	0.036	0.020	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.019	0.045	0.064	0.095	0.116	0.125	0.130	0.129	0.118	0.100	0.075	0.040	0.017	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.020	0.060	0.088	0.117	0.146	0.170	0.174	0.177	0.174	0.166	0.147	0.120	0.086	0.053	0.014	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.012	0.062	0.097	0.125	0.154	0.190	0.206	0.215	0.218	0.214	0.206	0.188	0.159	0.121	0.090	0.051	0.014	0.	0.	0.	0.
0.	0.	0.	0.014	0.052	0.100	0.144	0.175	0.205	0.245	0.261	0.272	0.273	0.268	0.255	0.237	0.208	0.177	0.139	0.095	0.050	0.014	0.	0.	0.
0.	0.	0.	0.025	0.078	0.128	0.173	0.218	0.256	0.291	0.305	0.310	0.317	0.311	0.295	0.276	0.247	0.217	0.176	0.127	0.082	0.033	0.	0.	0.
0.	0.	0.	0.047	0.111	0.160	0.206	0.255	0.300	0.338	0.356	0.367	0.366	0.359	0.346	0.325	0.296	0.257	0.215	0.169	0.114	0.061	0.014	0.	0.
0.	0.	0.013	0.072	0.130	0.186	0.241	0.295	0.340	0.380	0.400	0.409	0.417	0.411	0.393	0.364	0.333	0.287	0.239	0.189	0.137	0.075	0.028	0.	0.
0.	0.	0.022	0.079	0.141	0.203	0.263	0.314	0.359	0.408	0.448	0.467	0.466	0.453	0.437	0.400	0.360	0.315	0.267	0.211	0.152	0.100	0.044	0.	0.
0.	0.	0.040	0.098	0.157	0.219	0.269	0.322	0.379	0.433	0.472	0.495	0.500	0.490	0.456	0.413	0.368	0.320	0.265	0.214	0.161	0.105	0.043	0.	0.
0.	0.	0.049	0.106	0.158	0.209	0.262	0.309	0.371	0.430	0.474	0.502	0.520	0.498	0.459	0.420	0.372	0.325	0.280	0.225	0.171	0.108	0.042	0.	0.
0.	0.	0.046	0.105	0.160	0.206	0.258	0.303	0.364	0.420	0.462	0.491	0.502	0.486	0.449	0.408	0.367	0.317	0.277	0.219	0.163	0.100	0.043	0.	0.
0.	0.	0.037	0.094	0.149	0.195	0.239	0.284	0.339	0.397	0.439	0.463	0.471	0.455	0.425	0.382	0.346	0.300	0.256	0.205	0.146	0.086	0.023	0.	0.
0.	0.	0.032	0.083	0.134	0.177	0.219	0.260	0.309	0.355	0.396	0.428	0.430	0.421	0.391	0.353	0.313	0.269	0.228	0.188	0.123	0.072	0.013	0.	0.
0.	0.	0.005	0.060	0.110	0.151	0.194	0.232	0.268	0.312	0.350	0.373	0.383	0.374	0.350	0.310	0.278	0.236	0.194	0.156	0.110	0.040	0.	0.	0.
0.	0.	0.	0.030	0.084	0.121	0.158	0.200	0.234	0.271	0.304	0.326	0.330	0.325	0.306	0.279	0.245	0.207	0.169	0.128	0.079	0.030	0.	0.	0.
0.	0.	0.	0.026	0.064	0.107	0.144	0.185	0.216	0.245	0.267	0.285	0.287	0.280	0.267	0.241	0.203	0.173	0.136	0.054	0.045	0.009	0.	0.	0.
0.	0.	0.002	0.048	0.100	0.151	0.195	0.234	0.270	0.290	0.309	0.326	0.330	0.321	0.299	0.275	0.236	0.194	0.152	0.112	0.067	0.024	0.	0.	0.
0.	0.	0.025	0.075	0.131	0.187	0.224	0.265	0.308	0.347	0.379	0.401	0.403	0.387	0.363	0.331	0.291	0.240	0.192	0.140	0.095	0.041	0.	0.	0.
0.	0.	0.049	0.096	0.152	0.199	0.247	0.294	0.332	0.372	0.416	0.453	0.459	0.442	0.405	0.370	0.328	0.285	0.221	0.166	0.118	0.069	0.017	0.	0.
0.	0.	0.049	0.105	0.160	0.210	0.252	0.300	0.348	0.396	0.443	0.478	0.501	0.478	0.436	0.395	0.350	0.297	0.241	0.179	0.129	0.080	0.020	0.	0.
0.	0.	0.054	0.115	0.168	0.217	0.263	0.310	0.359	0.407	0.456	0.495	0.515	0.495	0.453	0.410	0.358	0.309	0.243	0.184	0.134	0.077	0.020	0.	0.
0.	0.	0.050	0.107	0.159	0.211	0.264	0.310	0.354	0.403	0.446	0.480	0.503	0.492	0.456	0.414	0.371	0.305	0.245	0.189	0.133	0.080	0.021	0.	0.
0.	0.	0.035	0.098	0.151	0.198	0.252	0.292	0.337	0.380	0.415	0.450	0.474	0.467	0.448	0.405	0.359	0.300	0.237	0.183	0.132	0.078	0.019	0.	0.
0.	0.	0.022	0.090	0.142	0.191	0.233	0.278	0.318	0.355	0.388	0.413	0.432	0.437	0.420	0.383	0.338	0.262	0.216	0.171	0.119	0.064	0.009	0.	0.
0.	0.	0.010	0.074	0.124	0.174	0.213	0.254	0.289	0.323	0.350	0.371	0.392	0.399	0.380	0.346	0.307	0.251	0.191	0.149	0.103	0.046	0.	0.	0.
0.	0.	0.003	0.052	0.109	0.150	0.189	0.221	0.257	0.284	0.309	0.323	0.345	0.345	0.326	0.301	0.261	0.212	0.164	0.115	0.073	0.026	0.	0.	0.
0.	0.	0.	0.019	0.074	0.124	0.164	0.193	0.223	0.252	0.273	0.297	0.304	0.304	0.286	0.258	0.220	0.180	0.134	0.097	0.052	0.	0.	0.	0.
0.	0.	0.	0.010	0.057	0.110	0.143	0.176	0.220	0.253	0.278	0.296	0.299	0.287	0.270	0.252	0.214	0.180	0.141	0.096	0.055	0.005	0.	0.	0.
0.	0.	0.	0.037	0.093	0.143	0.187	0.227	0.259	0.300	0.334	0.350	0.345	0.345	0.330	0.307	0.265	0.219	0.180	0.137	0.088	0.040	0.	0.	0.
0.	0.	0.007	0.064	0.114	0.166	0.209	0.254	0.307	0.353	0.387	0.408	0.419	0.419	0.395	0.369	0.324	0.280	0.224	0.172	0.117	0.065	0.013	0.	0.
0.	0.	0.031	0.079	0.132	0.181	0.231	0.282	0.340	0.396	0.442	0.470	0.491	0.477	0.448	0.408	0.365	0.320	0.267	0.199	0.142	0.087	0.029	0.	0.
0.	0.	0.030	0.083	0.142	0.190	0.242	0.300	0.364	0.412	0.464	0.500	0.518	0.500	0.466	0.423	0.379	0.332	0.270	0.212	0.151	0.100	0.038	0.	0.
0.	0.	0.027	0.090	0.147	0.200	0.251	0.314	0.369	0.423	0.466	0.504	0.534	0.511	0.475	0.432	0.388	0.343	0.284	0.218	0.160	0.100	0.047	0.	0.
0.	0.	0.037	0.096	0.151	0.204	0.260	0.324	0.386	0.431	0.473	0.510	0.533	0.519	0.484	0.442	0.403	0.355	0.293	0.225	0.174	0.122	0.063	0.	0.
0.	0.	0.032	0.088	0.141	0.188	0.243	0.304	0.358	0.404	0.441	0.475	0.490	0.478	0.445	0.412	0.373	0.320	0.264	0.203	0.152	0.098	0.041	0.	0.
0.	0.	0.023	0.077	0.124	0.175	0.225	0.275	0.325	0.378	0.413	0.439	0.452	0.443	0.427	0.400	0.359	0.307	0.252	0.200	0.150	0.095	0.043	0.	0.
0.	0.	0.003	0.043	0.094	0.144	0.194	0.237	0.280	0.324	0.361	0.379	0.381	0.380	0.369	0.344	0.310	0.259	0.213	0.165	0.118	0.061	0.014	0.	0.
0.	0.	0.	0.028	0.072	0.123	0.167	0.207	0.240	0.283	0.319	0.332	0.340	0.340	0.323	0.302	0.270	0.234	0.189	0.149	0.095	0.042	0.	0.	0.
0.	0.	0.	0.003	0.045	0.096	0.139	0.172	0.201	0.230	0.267	0.277	0.282	0.278	0.267	0.242	0.205	0.174	0.134	0.096	0.053	0.	0.	0.	0.
0.	0.	0.	0.005	0.045	0.096	0.144	0.191	0.232	0.263	0.283	0.303	0.304	0.300	0.287	0.267	0.237	0.193	0.149	0.098	0.050	0.	0.	0.	0.
0.	0.	0.	0.034	0.079	0.143	0.188	0.239	0.275	0.325	0.350	0.371	0.383	0.379	0.360	0.323	0.285	0.246	0.197	0.137	0.078	0.016	0.	0.	0.
0.	0.	0.009	0.063	0.116	0.176	0.223	0.268	0.318	0.362	0.402	0.430	0.442	0.434	0.405	0.366	0.324	0.280	0.230	0.178	0.113	0.043	0.005	0.	0.
0.	0.	0.026	0.085	0.138	0.189	0.240	0.289	0.337	0.386	0.425	0.468	0.494	0.473	0.434	0.390	0.350	0.304	0.256	0.205	0.138	0.062	0.019	0.	0.
0.	0.	0.033	0.090	0.144	0.195	0.240	0.289	0.337	0.383	0.422	0.477	0.509	0.490	0.447	0.402	0.358	0.313	0.270	0.217	0.147	0.078	0.028	0.	0.
0.	0.	0.038	0.097	0.155	0.192	0.239	0.288	0.339	0.395	0.436	0.480	0.504	0.483	0.444	0.405	0.357	0.311	0.267	0.214	0.147	0.080	0.026	0.	0.
0.	0.	0.036	0.086	0.143	0.187	0.238	0.280	0.334	0.384	0.428	0.467	0.482	0.482	0.442	0.402	0.353	0.313	0.265	0.215	0.141	0.082	0.023	0.	0.
0.	0.	0.015	0.070	0.116	0.174	0.221	0.276	0.320	0.364	0.406	0.430	0.444	0.437	0.412	0.374	0.341	0.295	0.245	0.192	0.125	0.062	0.008	0.	0.
0.	0.	0.	0.042	0.091	0.152	0.200	0.253	0.295	0.329	0.371	0.387	0.400	0.385	0.367	0.339	0.304	0.262	0.214	0.159	0.098	0.037	0.	0.	0.
0.	0.	0.	0.020	0.071	0.125	0.169	0.216	0.250	0.285	0.317	0.332	0.339	0.330	0.312	0.288	0.254	0.220	0.169	0.115	0.059	0.011	0.	0.	0.
0.	0.	0.	0.005	0.044	0.093	0.146	0.186	0.214	0.242	0.270	0.281	0.290	0.283	0.272	0.250	0.215	0.181	0.131	0.079	0.037	0.	0.	0.	0.
0.	0.	0.	0.	0.023	0.065	0.105	0.138	0.174	0.202	0.219	0.230	0.240	0.227	0.224	0.205	0.175	0.133	0.100	0.049	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.028	0.067	0.098	0.132	0.154	0.173	0.189	0.189	0.187	0.186	0.158	0.136	0.100	0.060	0.022	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.004	0.025	0.060	0.086	0.109	0.127	0.137	0.135	0.130	0.124	0.108	0.082	0.056	0.022	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.013	0.037	0.059	0.075	0.085	0.084	0.079	0.069	0.058	0.038	0.012	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.006	0.014	0.024	0.027	0.027	0.024	0.015	0.013	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Disturbed Stockpile Depth Measurements

MEASUREMENT LOCATIONS (25)

CROSS SECTIONS (61)

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.015	0.015	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.023	0.048	0.054	0.059	0.065	0.061	0.038	0.031	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.024	0.044	0.048	0.063	0.090	0.113	0.117	0.122	0.112	0.090	0.068	0.019	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.037	0.067	0.101	0.129	0.149	0.162	0.168	0.175	0.162	0.132	0.098	0.068	0.040	0.040	0.029	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.015	0.031	0.066	0.107	0.149	0.185	0.211	0.221	0.221	0.190	0.162	0.137	0.102	0.080	0.087	0.053	0.023	0.	0.	0.	0.
0.	0.	0.	0.025	0.052	0.071	0.105	0.142	0.180	0.217	0.251	0.265	0.262	0.232	0.204	0.173	0.139	0.124	0.102	0.057	0.025	0.	0.	0.	0.
0.	0.	0.	0.015	0.024	0.060	0.107	0.144	0.184	0.226	0.267	0.302	0.294	0.265	0.231	0.200	0.173	0.136	0.090	0.043	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.015	0.062	0.110	0.160	0.208	0.251	0.300	0.333	0.339	0.305	0.273	0.233	0.191	0.146	0.095	0.057	0.015	0.	0.	0.	0.
0.	0.	0.	0.	0.018	0.070	0.119	0.169	0.214	0.266	0.312	0.363	0.379	0.340	0.295	0.242	0.202	0.156	0.119	0.076	0.046	0.027	0.	0.	0.
0.	0.	0.	0.015	0.037	0.084	0.131	0.173	0.222	0.272	0.325	0.377	0.410	0.360	0.312	0.264	0.215	0.173	0.136	0.090	0.046	0.025	0.	0.	0.
0.	0.	0.	0.010	0.058	0.100	0.138	0.179	0.225	0.273	0.322	0.374	0.424	0.359	0.310	0.270	0.225	0.182	0.134	0.078	0.020	0.	0.	0.	0.
0.	0.	0.	0.	0.042	0.093	0.142	0.187	0.234	0.284	0.335	0.391	0.447	0.380	0.333	0.287	0.227	0.190	0.150	0.098	0.037	0.	0.	0.	0.
0.	0.	0.	0.006	0.054	0.098	0.138	0.184	0.231	0.287	0.336	0.400	0.425	0.377	0.328	0.282	0.235	0.185	0.146	0.092	0.042	0.	0.	0.	0.
0.	0.	0.	0.020	0.065	0.112	0.158	0.203	0.245	0.293	0.349	0.403	0.436	0.390	0.346	0.300	0.251	0.205	0.161	0.122	0.079	0.028	0.	0.	0.
0.	0.	0.	0.011	0.065	0.105	0.145	0.190	0.239	0.289	0.342	0.395	0.418	0.381	0.335	0.287	0.240	0.195	0.151	0.112	0.072	0.020	0.	0.	0.
0.	0.	0.	0.015	0.055	0.092	0.140	0.190	0.235	0.285	0.341	0.378	0.390	0.380	0.340	0.294	0.242	0.196	0.152	0.112	0.069	0.028	0.	0.	0.
0.	0.	0.	0.	0.035	0.075	0.119	0.167	0.213	0.263	0.292	0.315	0.316	0.315	0.302	0.266	0.220	0.168	0.135	0.093	0.066	0.016	0.	0.	0.
0.	0.	0.	0.015	0.052	0.087	0.131	0.173	0.215	0.250	0.275	0.291	0.293	0.292	0.276	0.250	0.222	0.189	0.142	0.110	0.068	0.	0.	0.	0.
0.	0.	0.	0.015	0.042	0.059	0.131	0.174	0.216	0.265	0.313	0.335	0.333	0.325	0.305	0.258	0.212	0.170	0.120	0.075	0.033	0.	0.	0.	0.
0.	0.	0.	0.030	0.067	0.102	0.140	0.187	0.231	0.273	0.320	0.374	0.402	0.379	0.323	0.275	0.224	0.171	0.125	0.085	0.032	0.	0.	0.	0.
0.	0.	0.015	0.039	0.071	0.098	0.135	0.177	0.224	0.271	0.315	0.365	0.417	0.360	0.303	0.253	0.207	0.156	0.110	0.062	0.019	0.	0.	0.	0.
0.	0.	0.015	0.045	0.058	0.090	0.135	0.183	0.225	0.274	0.322	0.375	0.442	0.386	0.324	0.275	0.222	0.174	0.133	0.084	0.030	0.	0.	0.	0.
0.	0.	0.	0.005	0.027	0.073	0.117	0.162	0.211	0.262	0.319	0.380	0.441	0.380	0.319	0.269	0.215	0.170	0.126	0.079	0.030	0.	0.	0.	0.
0.	0.	0.	0.	0.031	0.078	0.128	0.171	0.218	0.271	0.321	0.385	0.456	0.405	0.340	0.290	0.248	0.202	0.156	0.110	0.072	0.053	0.025	0.	0.
0.	0.	0.	0.	0.015	0.080	0.118	0.165	0.209	0.260	0.310	0.365	0.423	0.391	0.335	0.283	0.240	0.200	0.160	0.128	0.093	0.064	0.018	0.	0.
0.	0.	0.	0.	0.050	0.090	0.138	0.188	0.232	0.278	0.328	0.375	0.429	0.427	0.365	0.315	0.270	0.229	0.186	0.146	0.100	0.052	0.	0.	0.
0.	0.	0.	0.005	0.042	0.079	0.124	0.173	0.220	0.261	0.305	0.358	0.382	0.390	0.349	0.305	0.251	0.208	0.168	0.122	0.075	0.025	0.	0.	0.
0.	0.	0.	0.033	0.075	0.110	0.150	0.187	0.225	0.265	0.305	0.338	0.353	0.355	0.335	0.301	0.250	0.200	0.152	0.112	0.062	0.	0.	0.	0.
0.	0.	0.	0.005	0.035	0.079	0.125	0.160	0.200	0.235	0.260	0.282	0.291	0.290	0.271	0.245	0.208	0.171	0.128	0.085	0.030	0.	0.	0.	0.
0.	0.	0.	0.020	0.059	0.105	0.142	0.179	0.219	0.258	0.285	0.302	0.305	0.295	0.277	0.260	0.225	0.183	0.142	0.095	0.040	0.	0.	0.	0.
0.	0.	0.	0.	0.019	0.061	0.118	0.163	0.219	0.268	0.321	0.345	0.348	0.343	0.315	0.269	0.223	0.173	0.133	0.100	0.060	0.020	0.	0.	0.
0.	0.	0.	0.	0.035	0.085	0.135	0.172	0.228	0.285	0.339	0.392	0.417	0.378	0.333	0.292	0.249	0.203	0.165	0.125	0.095	0.060	0.026	0.	0.
0.	0.	0.	0.005	0.030	0.075	0.122	0.172	0.220	0.277	0.325	0.375	0.405	0.360	0.310	0.264	0.215	0.172	0.128	0.089	0.060	0.036	0.	0.	0.
0.	0.	0.003	0.035	0.080	0.111	0.140	0.182	0.225	0.265	0.317	0.365	0.412	0.362	0.315	0.260	0.215	0.168	0.119	0.072	0.022	0.	0.	0.	0.
0.	0.	0.	0.023	0.059	0.102	0.148	0.178	0.223	0.264	0.308	0.362	0.419	0.377	0.329	0.271	0.228	0.180	0.130	0.080	0.023	0.	0.	0.	0.
0.	0.	0.	0.010	0.046	0.090	0.132	0.170	0.219	0.260	0.312	0.362	0.415	0.382	0.321	0.272	0.220	0.172	0.125	0.078	0.025	0.	0.	0.	0.
0.	0.	0.	0.021	0.055	0.098	0.140	0.177	0.228	0.270	0.310	0.360	0.414	0.395	0.340	0.288	0.239	0.190	0.151	0.104	0.073	0.059	0.019	0.	0.
0.	0.	0.	0.035	0.075	0.108	0.141	0.176	0.222	0.266	0.310	0.359	0.402	0.391	0.338	0.289	0.240	0.191	0.150	0.109	0.075	0.040	0.	0.	0.
0.	0.	0.015	0.050	0.062	0.100	0.139	0.171	0.218	0.264	0.305	0.351	0.386	0.385	0.339	0.287	0.241	0.190	0.143	0.105	0.060	0.020	0.	0.	0.
0.	0.	0.	0.005	0.025	0.069	0.120	0.164	0.206	0.248	0.296	0.325	0.334	0.335	0.310	0.268	0.211	0.168	0.120	0.078	0.038	0.	0.	0.	0.
0.	0.	0.	0.	0.015	0.065	0.109	0.158	0.191	0.232	0.257	0.284	0.290	0.282	0.270	0.243	0.200	0.158	0.114	0.070	0.031	0.	0.	0.	0.
0.	0.	0.	0.	0.031	0.084	0.125	0.163	0.209	0.257	0.286	0.305	0.309	0.307	0.285	0.242	0.200	0.158	0.111	0.071	0.015	0.	0.	0.	0.
0.	0.	0.	0.019	0.060	0.095	0.133	0.165	0.207	0.252	0.312	0.362	0.380	0.350	0.300	0.252	0.209	0.169	0.130	0.087	0.045	0.028	0.	0.	0.
0.	0.	0.	0.005	0.052	0.092	0.128	0.158	0.200	0.252	0.307	0.360	0.400	0.342	0.300	0.253	0.210	0.168	0.124	0.091	0.059	0.025	0.	0.	0.
0.	0.	0.	0.	0.048	0.090	0.127	0.159	0.205	0.259	0.310	0.366	0.399	0.350	0.309	0.260	0.212	0.173	0.134	0.082	0.042	0.	0.	0.	0.
0.	0.	0.	0.010	0.046	0.080	0.123	0.168	0.212	0.265	0.322	0.369	0.402	0.358	0.307	0.254	0.207	0.168	0.121	0.075	0.031	0.	0.	0.	0.
0.	0.	0.	0.	0.031	0.070	0.119	0.168	0.207	0.266	0.315	0.365	0.410	0.359	0.310	0.250	0.202	0.157	0.110	0.070	0.030	0.	0.	0.	0.
0.	0.	0.	0.	0.045	0.088	0.131	0.172	0.222	0.270	0.316	0.366	0.390	0.344	0.288	0.236	0.190	0.145	0.115	0.085	0.050	0.022	0.	0.	0.
0.	0.	0.013	0.037	0.065	0.090	0.130	0.175	0.220	0.265	0.309	0.343	0.350	0.320	0.275	0.232	0.178	0.137	0.088	0.060	0.045	0.032	0.022	0.	0.
0.	0.	0.024	0.052	0.060	0.080	0.115	0.165	0.214	0.252	0.289	0.303	0.308	0.290	0.256	0.214	0.173	0.127	0.085	0.041	0.020	0.	0.	0.	0.
0.	0.	0.026	0.050	0.045	0.061	0.108	0.151	0.192	0.234	0.252	0.261	0.265	0.254	0.220	0.190	0.154	0.109	0.067	0.028	0.	0.	0.	0.	0.
0.	0.	0.014	0.026	0.020	0.059	0.092	0.137	0.179	0.205	0.214	0.223	0.220	0.217	0.190	0.159	0.128	0.094	0.050	0.012	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.037	0.083	0.125	0.163	0.171	0.170	0.178	0.174	0.182	0.174	0.131	0.098	0.067	0.031	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.020	0.067	0.108	0.135	0.128	0.125	0.130	0.130	0.145	0.153	0.103	0.060	0.027	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.012	0.026	0.060	0.102	0.094	0.075	0.077	0.082	0.104	0.122	0.080	0.028	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.028	0.064	0.100	0.094	0.074	0.073	0.080	0.102	0.123	0.078	0.028	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.015	0.030	0.027	0.015	0.	0.015	0.019	0.025	0.017	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

COMPUTER PROGRAM TO DETERMINE THE APPROXIMATE VOLUME OF A BANKED STOCKPILE ASSUMING THE STOCKPILE CROSS SECTIONS ARE ISOSCELES TRIANGLES.

Purpose

The purpose of this program is to determine the approximate volume of a stockpile whose sides are in contact with retaining walls. A secondary purpose is to determine the angle of repose of the material in the stockpile.

Language

FORTRAN IV (IBM 7040 COMPUTER)

Symbolic Dictionary

VARIABLE	S/A*	I/O**	DESCRIPTION
PW	S	I	Distance between apex of the stockpile and the retaining wall.
WF	A	--	Width of the small triangular section cut off by the retaining wall.
XR	A	I	Distance the base of the pile is from right boundary line.
XL	A	I	Distance the base of the pile is from the left boundary line.
M	S	I/O	Number of measurement points.
L	S	I/O	Distance between the end boundary lines.
W	S	I/O	Distance between the side boundary lines.
ALPHA	S	I/O	Inclination angle of the stockpile - same as apparent angle of repose.

*S - Single variable; A - Array of variables

**I - Input; O - Output

ALPHAR	S	--	Alpha expressed in radians.
N	S	I	Number of measurement stations.
VACT	S	I/O	Actual volume of the stockpile.
DELTAL	S	O	Length interval size.
BASE	S	--	Width of stockpile at the base.
AREA	A	O	Area of a cross section.
SUMA	S	--	A holding variable used to accumulate the areas of cross sections for calculating the volume by the Trapezoidal Rule.
VCALTR	S	O	The calculated volume of the stockpile utilizing the Trapezoidal Rule.
PERRTR	S	O	Per cent error of the calculated volume using the Trapezoidal Rule.
ODAREA	S	--	A holding variable used to accumulate the areas of the odd numbered stockpile cross sections for calculating the stockpile volume using Simpson's Rule.
EVAREA	S	--	A holding variable used to accumulate the areas of the even numbered cross sections for calculating the stockpile volume using Simpson's Rule.
VCALSN	S	O	The calculated stockpile volume using Simpson's Rule.
PERRSN	S	O	Per cent error of the calculated volume using Simpson's Rule.
SUMTWA	S	--	A holding variable used to accumulate the weighted areas of cross sections for calculating the stockpile volume using Weddle's Rule.
VCALWD	S	O	The calculated stockpile volume using Weddle's Rule.
PERRWD	S	O	Per cent error of the calculated volume using Weddle's Rule.

Program Routine

The program utilizes as data points the distances which the stockpile cross sections are from the side boundary lines. The area of each cross section is then computed assuming that it is an isosceles triangle. The program checks to see if the stockpile is in contact with the retaining wall. If it is, then the program calculates the net area of the cross section by subtracting the area of the cut-off triangular section from the gross area of the standard isosceles triangles.

These areas are then integrated using the Trapezoidal, Simpson's and Weddle's quadrature formulas for a set interval size. The error in the calculated volume is then computed.

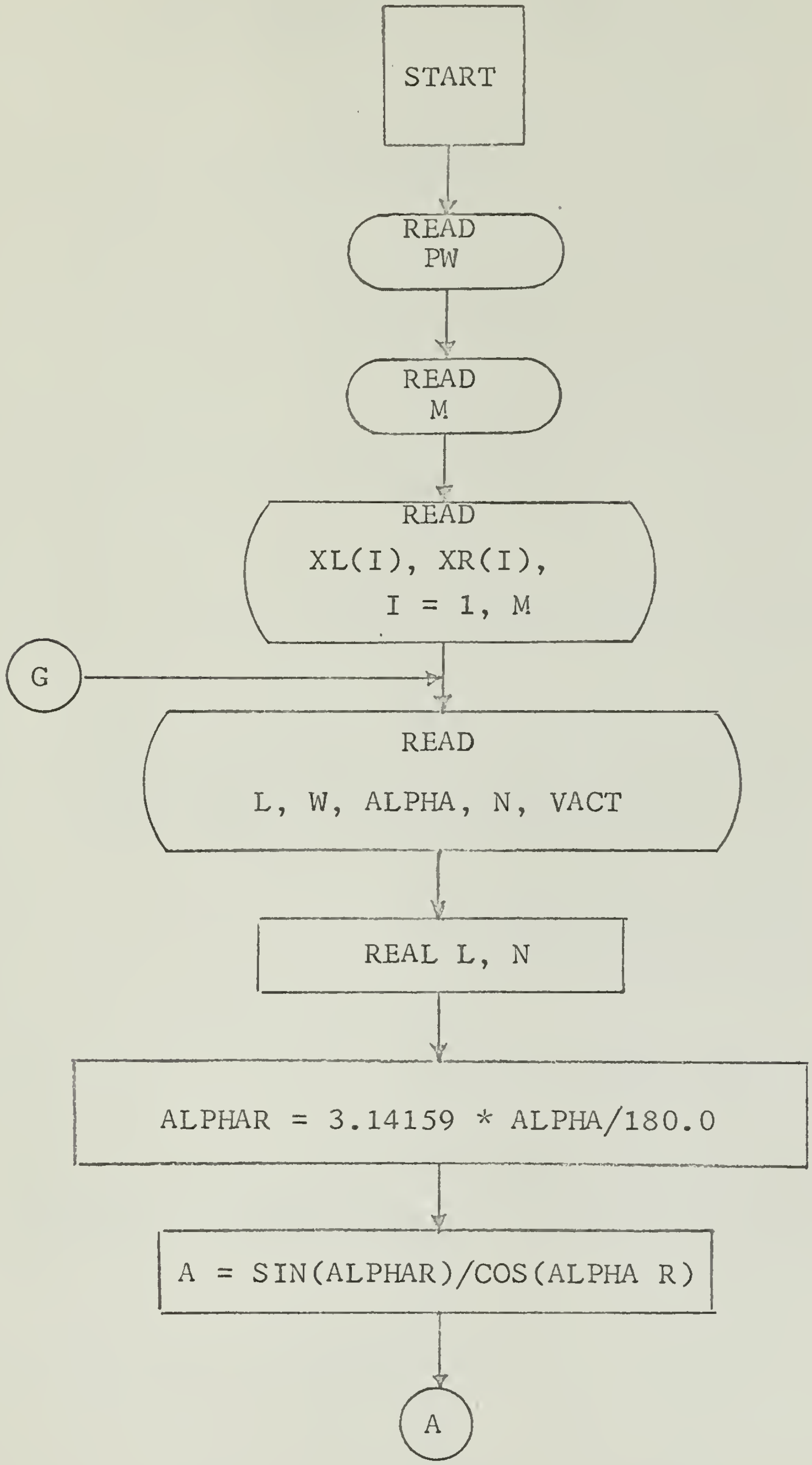
The angle of repose is determined when the calculated volume error is a zero value based upon varying the parameter, ALPHA.

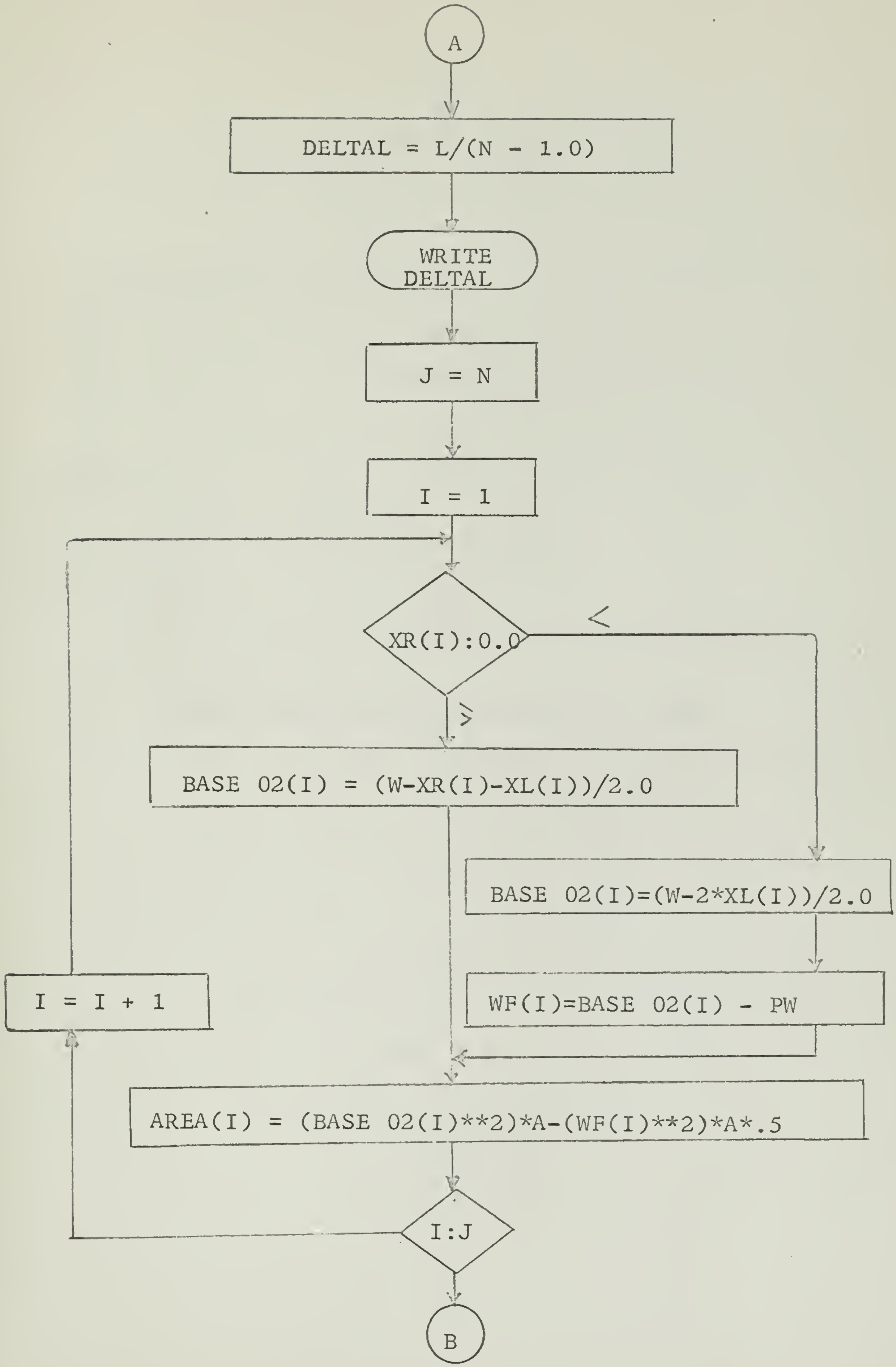
```
C     THIS PROGRAM DETERMINES THE APPROXIMATE VOLUME OF A STOCKPILE
C     WHICH IS BANKED AGAINST A RETAINING WALL AND FURTHER
C     ASSUMING THAT THE CROSS SECTIONS ARE ISOSCELES TRIANGLES.
C
      DIMENSION  XL(100), XR(100), AREA(100), BASEO2(100), WF(100)
      READ(5,26)  PW
   26 FORMAT(F5.1)
      READ (5,5)  M
    5 FORMAT (I2)
      WRITE ( 6,6)
    6 FORMAT (15X, 20HM = NO. OF DATA PTS.//)
      WRITE (6,7)  M
    7 FORMAT (19X, I2//)
      READ (5,20) (XL(I) , XR(I) , I = 1, M)
   20 FORMAT (2F10.2)
      WRITE (6,11)
   11 FORMAT (15X, 17HINPUT DATA POINTS//)
      WRITE (6,13)
   13 FORMAT (15X, 3HNO., 15X, 2HXL, 15X, 2HXR//)
      WRITE (6,12) ( I, XL(I), XR(I) , I = 1, M)
   12 FORMAT ( 15X, I2, 10X, F10.2,  8X, F10.2//)
    9 READ (5,10) L, W, ALPHA, N, VACT
   10 FORMAT (2F10.2,F6.4, F6.0, F10.2)
      WRITE (6,14)
   14 FORMAT (15X, 20HINPUT DATA CONSTANTS//)
      WRITE (6,16)
   16 FORMAT(15X, 1HL, 10X, 1HW, 10X, 5HALPHA, 10X, 1HN, 10X, 4HVACT//)
      WRITE (6,17) L, W, ALPHA, N, VACT
   17 FORMAT(13X, F5.2, 6X, F5.2, 8X, F5.2, 8X, F5.0, 9X, F10.2//)
C
C     ASSUME CROSS SECTIONS ARE ISOSCELES TRIANGLES FOR COMPUTING AREAS.
C
      REAL L,N
      ALPHAR = 3.14159 * ALPHA / 180.0
      A = SIN(ALPHAR)/COS(ALPHAR)
      DELTAL = L/(N-1.0)
      WRITE (6,18)
   18 FORMAT (15X, 17HINTERVAL = DELTAL//)
      WRITE (6,19)  DELTAL
   19 FORMAT (22X, F5.2//)
      J = N
      DO 44  I = 1,J
      IF(XR(I).LE.0.0) GO TO 33
   22 BASEO2(I) = (W-XR(I)-XL(I))/2.0
      AREA(I) = (BASEO2(I)**2)*A
      GO TO 44
   33 BASEO2(I) = (W-2.0*XL(I)) /2.0
      WF(I) = BASEO2(I) -PW
  100  AREA(I) = (BASEO2(I)**2)*A - (WF(I)**2)*A*0.5
      J = I
   44 CONTINUE
      AREA(1) = 0.0
      AREA(J) = 0.0
      WRITE (6,500)
  500 FORMAT (15X, 22HAREA OF CROSS SECTIONS//)
```

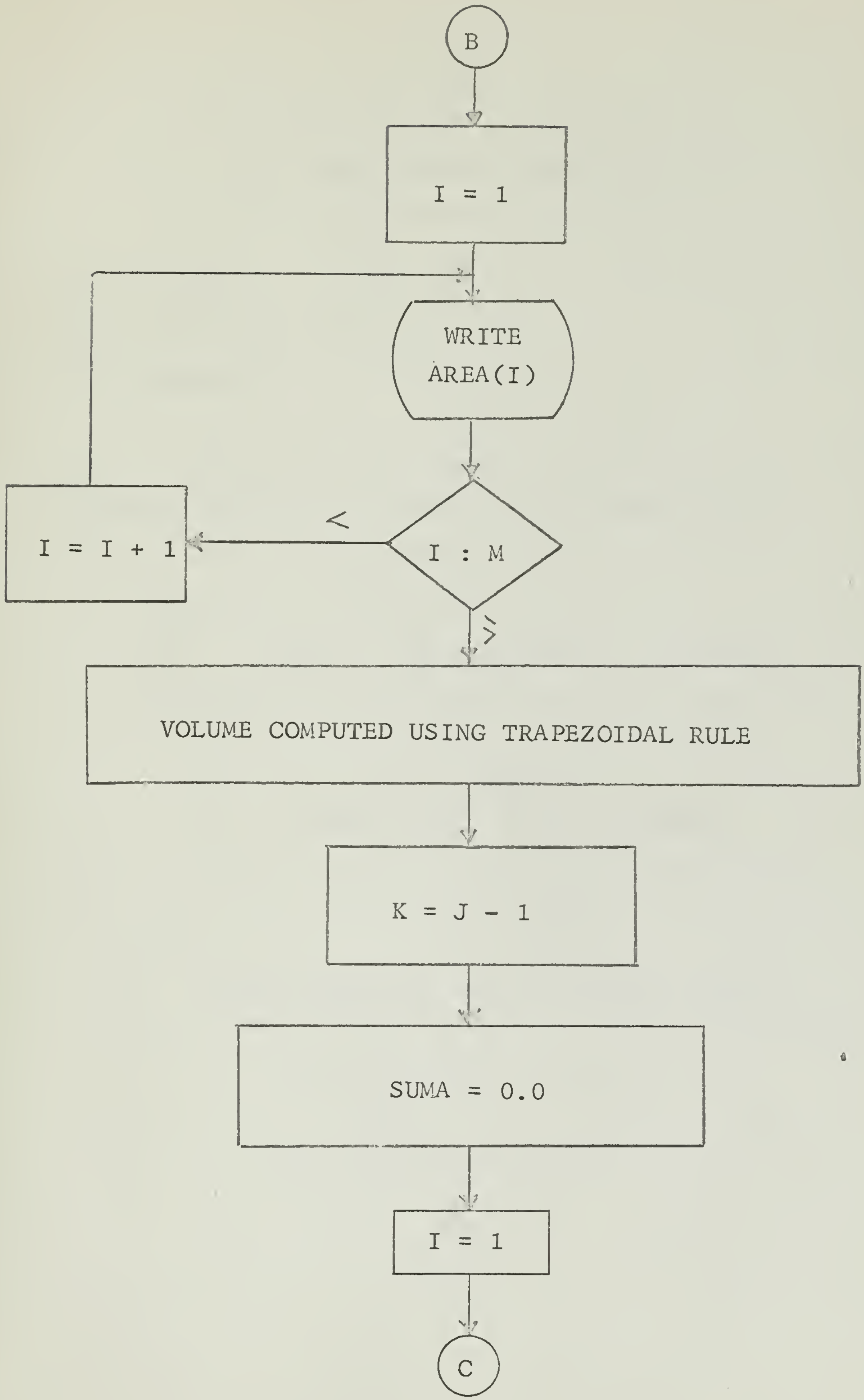
[illegible]

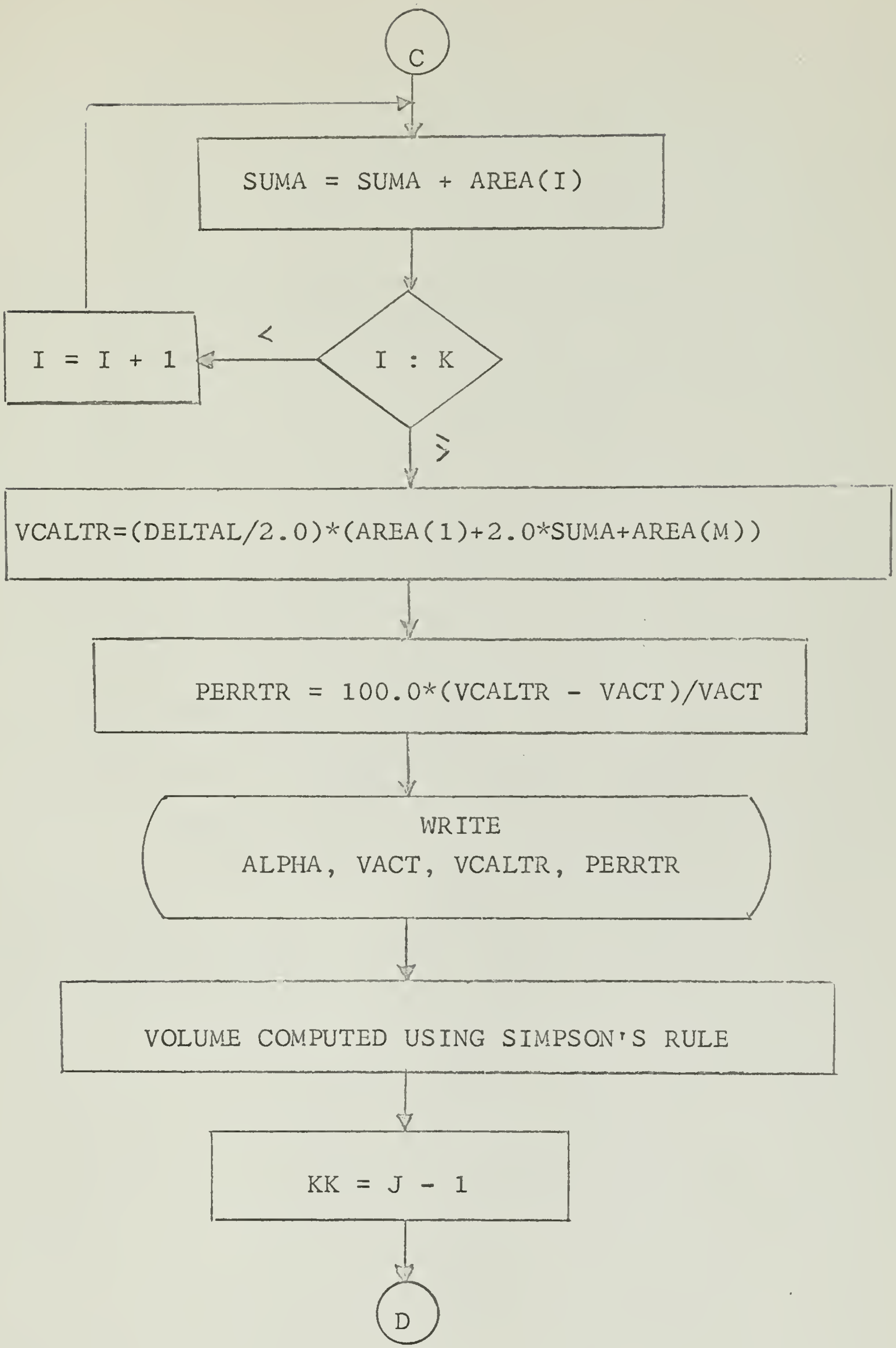
```
      WRITE (6,600) (I, AREA(I), I = 1, M)
  600 FORMAT(15X, I2, F10.4)
C     STOCKPILE VOLUME COMPUTED WITH THE TRAPEZOID INTEGRATION RULE.
      K = J - 1
      SUMA = 0.0
      DO 800 I = 2, K
  800 SUMA = SUMA + AREA(I)
      VCALTR = (DELTAL/2.0) * (AREA(1) + 2.0 *SUMA + AREA(M))
      PERRTR = 100.0 * (VCALTR - VACT) / VACT
      WRITE ( 6,900)
  900 FORMAT(15X, 5HALPHA, 15X, 4HVACT, 15X, 6HVCALTR, 15X, 6HPERRTR//)
      WRITE (6,400) ALPHA, VACT,  VCALTR,  PERRTR
  400 FORMAT (15X, F5.2, 15X, F7.2, 12X, F7.2, 13X, F6.2//)
C
C     STOCKPILE VOLUME COMPUTED WITHSIMPSONS INTEGRATION RULE.
      KK = J - 1
      KKK = KK - 1
      ODAREA = 0.0
      EVAREA = 0.0
      DO 910 I = 2, KK, 2
  910 EVAREA = EVAREA + AREA (I)
      DO 920 I = 3, KKK, 2
  920 ODAREA = ODAREA + AREA (I)
      VCALSN = (DELTAL/3.)*(AREA(1) + 4.*EVAREA + 2.*ODAREA + AREA(M))
      PERRSN = 100.0 * (VCALSN - VACT) / VACT
      WRITE ( 6, 950)
  950 FORMAT(15X, 5HALPHA, 15X, 4HVACT, 15X, 6HVCALSN, 15X, 6HPERRSN//)
      WRITE (6,400)  ALPHA,  VACT,  VCALSN,  PERRSN
C
C     STOCKPILE VOLUME COMPUTED WITHWEDDLES INTEGRATION RULE.
      SUMTWA = 0.0
      DO 960  I = 2, M, 6
  960 SUMTWA = SUMTWA + 1.*AREA(I-1) + 5.*AREA(I) + 1.*AREA(I+1) +
     16.*AREA(I+2) + 1.*AREA(I+3) + 5.*AREA(I+4) + 1.*AREA(I+5)
      VCALWD = .3*DELTAL*SUMTWA
      PERRWD = 100.0*(VCALWD - VACT)/VACT
      WRITE (6,970)
  970 FORMAT(15X, 5HALPHA, 15X, 4HVACT, 15X, 6HVCALWD, 15X, 6HPERRWD//)
      WRITE (6,400) ALPHA, VACT, VCALWD, PERRWD
      GO TO 9
      CALL EXIT
      END
```

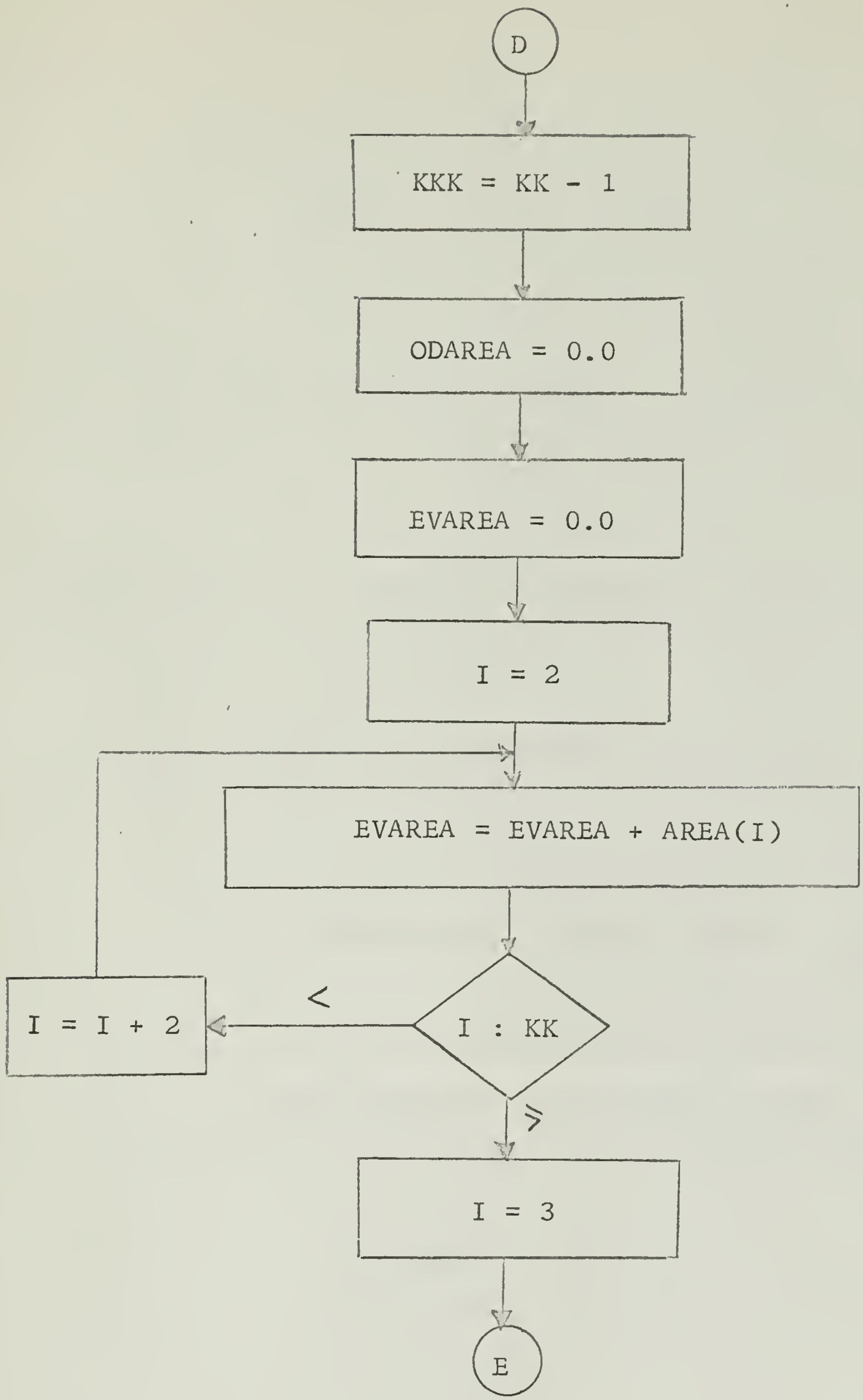
FLOW DIAGRAM FOR COMPUTER PROGRAM TO DETERMINE THE APPROXIMATE VOLUME OF A BANKED STOCKPILE ASSUMING THE CROSS SECTIONS TO BE ISOSCELES TRIANGLES.

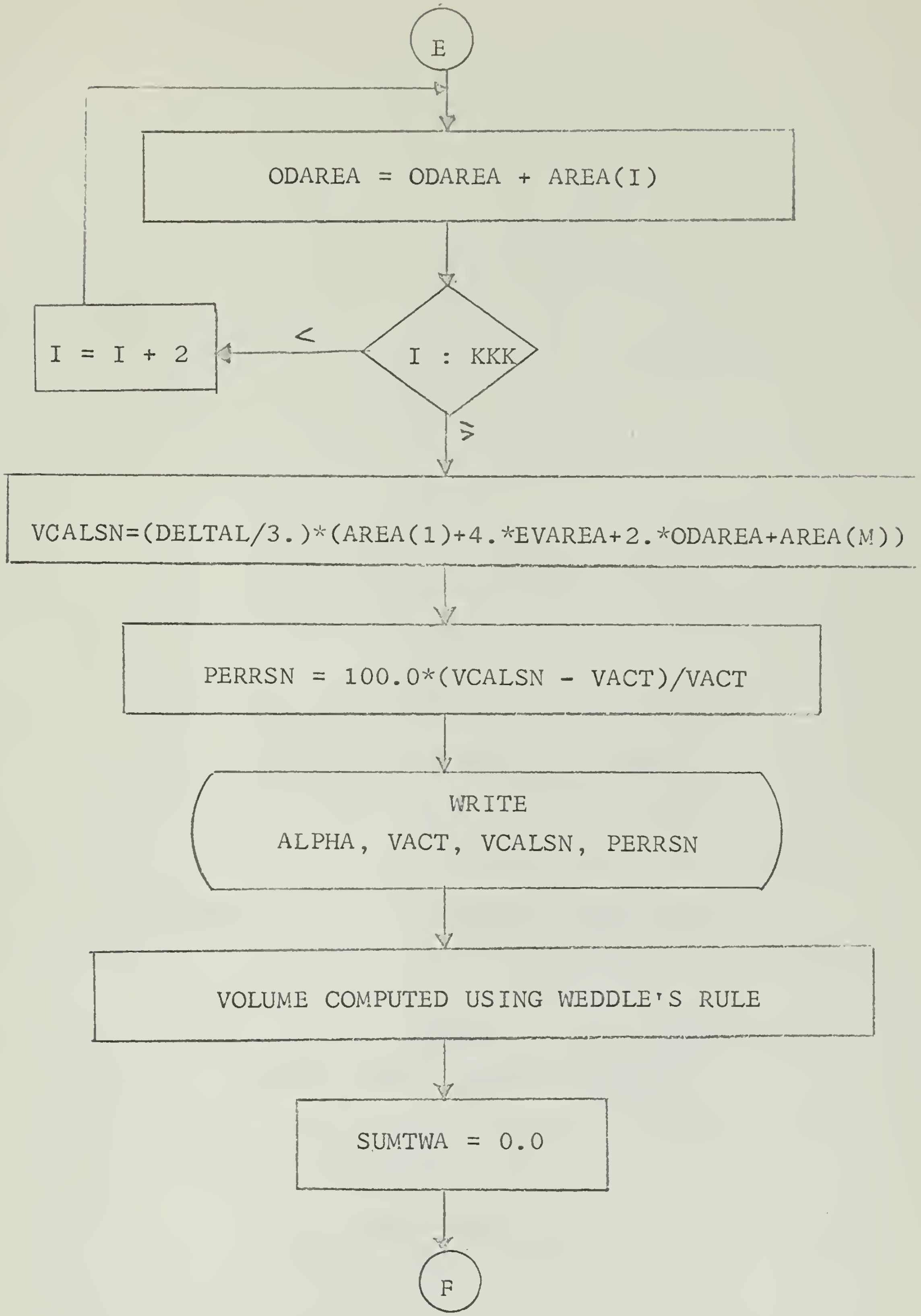


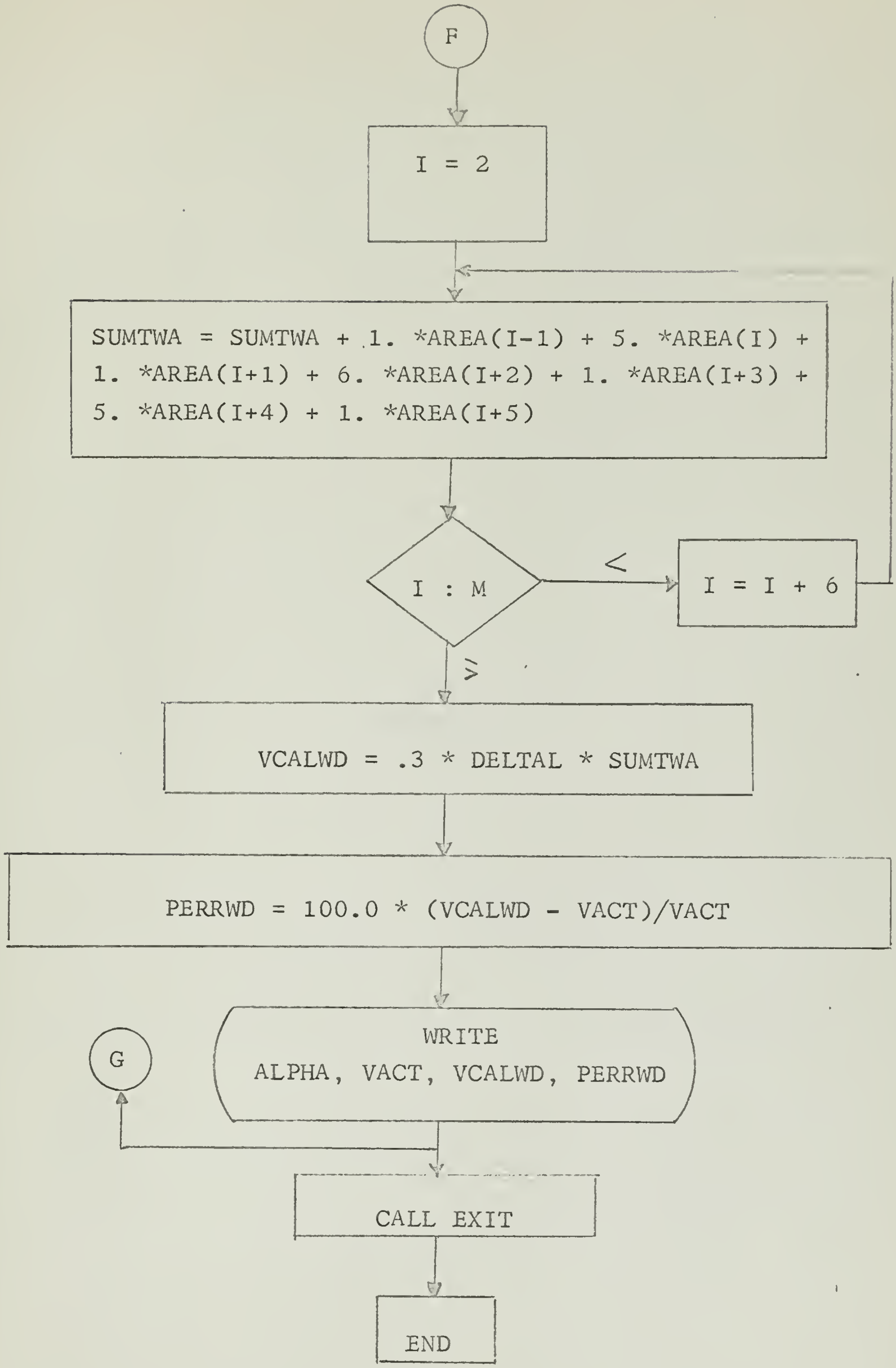

A
DELTAL = L/(N - 1.0)
WRITE DELTAL
J = N
I = 1
XR(I):0.0
<
>
BASE 02(I) = (W-XR(I)-XL(I))/2.0
BASE 02(I)=(W-2*XL(I))/2.0
WF(I)=BASE 02(I) - PW
I = I + 1
AREA(I) = (BASE 02(I)**2)*A-(WF(I)**2)*A*.5
I:J
B


B
I = 1
WRITE AREA(I)
I = I + 1
<
I : M
≥
VOLUME COMPUTED USING TRAPEZOIDAL RULE
K = J - 1
SUMA = 0.0
I = 1
C


C
SUMA = SUMA + AREA(I)
I : K
<
I = I + 1
≥
VCALTR=(DELTAL/2.0)*(AREA(1)+2.0*SUMA+AREA(M))
PERRTR = 100.0*(VCALTR - VACT)/VACT
WRITE
ALPHA, VACT, VCALTR, PERRTR
VOLUME COMPUTED USING SIMPSON'S RULE
KK = J - 1
D


D
KKK = KK - 1
ODAREA = 0.0
EVAREA = 0.0
I = 2
EVAREA = EVAREA + AREA(I)
I = I + 2
<
I : KK
≥
I = 3
E


E
ODAREA = ODAREA + AREA(I)
I = I + 2
<
I : KKK
≥
VCALSN=(DELTAL/3.)*(AREA(1)+4.*EVAREA+2.*ODAREA+AREA(M))
PERRSN = 100.0*(VCALSN - VACT)/VACT
WRITE
ALPHA, VACT, VCALSN, PERRSN
VOLUME COMPUTED USING WEDDLE'S RULE
SUMTWA = 0.0
F


F
I = 2
SUMTWA = SUMTWA + 1. *AREA(I-1) + 5. *AREA(I) + 1. *AREA(I+1) + 6. *AREA(I+2) + 1. *AREA(I+3) + 5. *AREA(I+4) + 1. *AREA(I+5)
I : M
<
I = I + 6
≥
VCALWD = .3 * DELTAL * SUMTWA
PERRWD = 100.0 * (VCALWD - VACT)/VACT
WRITE
ALPHA, VACT, VCALWD, PERRWD
G
CALL EXIT
END

APPENDIX B

FIGURE I

CALCULATION OF STOCKPILE VOLUME FROM BASE MEASUREMENTS

Boundary Width BW = ft.	Angle of Repose α, =	Tan α =	C = Tan α /4.0 C =	Interval Size, I =BL/(# of Cross Sect. -1) =	Boundary Length BL = ft.

Col. (1) Cross Section	Col. (2) Distance to Pile Base from Rt. Side Ft.	Col. (3) Distance to Pile Base from Lt. Side Ft.	Col. (4) Col(2) + Col(3) Ft.	Col. (5) Width of Pile Base BW - Col (4) Ft.	Col. (6) Col (5) x Col (5) Ft^2	Col. (7) Cross Sect. Area Col (6) x C Ft^2		Col. (8) (ODD CRS.SCT) x 2 Col (7) ODD x 2	Col. (9) (EVEN CRS.SCT) x 4 Col (7) EVEN x 4
TOTAL									

ESTIMATE USING TRAPEZOID RULE	ESTIMATE USING SIMPSON'S RULE
Volume = Interval x Total Col (7) =	Volume = Interval x Total Col (8) + Total Col (9) =
Tonnage = Volume x Bulk Density x $\left(\frac{1}{2000}\right)$ =	Tonnage = Volume x Bulk Density x $\left(\frac{1}{2000}\right)$ =

FIGURE 2

CALCULATION OF STOCKPILE VOLUME FROM BASE MEASUREMENTS

Boundary Width BW = 22.0 ft.	Angle of Repose α = 29.7	Tan α = .57	C = Tan α /4.0 C = .14	Interval Size I = BL/(# of Cross Sect. − 1) = 1.0	Boundary Length BL = 24.0 ft.

Col.(1) Cross Section	Col.(2) Distance to Pile Base from Rt. Side Ft.	Col.(3) Distance to Pile Base from Lt. Side Ft.	Col.(4) Col(2) + Col(3) Ft.	Col.(5) Width of Pile Base BW − Col (4) Ft.	Col.(6) Col(5) x Col(5) Ft.^{2}	Col.(7) Cross Sect. Area Col(6) x C Ft.^{2}		Col.(8) (ODD CRSECT) x2 Col(7) ODD x 2	Col.(9) (EVEN CRSECT) x4 Col(7) EVEN x 4
1	0.0	22.0	22.0	0.0	0.0	0.0		0	------
2	2.0	2.0	18.0	4.0	16.0	2.24		----	8.96
3	6.6	6.5	13.1	8.9	79.2	11.09		22.18	-----
4	4.9	5.2	10.1	11.9	141.6	19.82		-----	79.28
5	3.8	4.0	7.8	14.2	201.6	28.22		56.44	-----
6	2.9	3.3	6.2	15.8	249.6	34.94		-----	139.76
7	2.2	2.6	4.8	17.2	295.8	41.41		82.82	-----
8	1.6	2.1	3.7	18.3	334.9	46.89		-----	187.56
9	1.1	1.6	2.7	19.3	372.5	52.15		104.30	-----
10	.8	1.2	2.0	20.0	400.0	56.00		-----	224.00
11	.5	1.0	1.5	20.5	420.3	58.84		117.68	------
12	.3	.8	1.1	20.9	436.8	61.15		-----	244.60
13	.4	.7	1.1	20.9	436.8	61.15		122.30	-----
14	.5	.8	1.3	20.7	428.5	59.99		------	239.96
15	.6	1.0	1.6	20.4	416.2	58.27		116.54	-----
16	.8	1.3	2.1	19.9	396.0	55.44		------	221.76
17	1.3	1.7	3.0	19.0	361.0	50.54		101.08	------
18	1.7	2.1	3.8	18.2	331.2	46.37		-----	185.48
19	2.4	2.8	5.2	16.8	282.2	39.51		79.02	-------
20	3.3	3.5	6.8	15.2	231.0	32.34		-----	129.36
21	4.4	4.5	8.9	13.1	171.6	24.02		48.04	------
22	6.0	6.4	12.4	9.6	92.2	12.91		-----	51.64
23	9.2	10.0	19.2	2.8	7.8	1.09		2.18	------
24	0.0	22.0	22.0	0.0	0.0	0.0		----	------
25	0.0	22.0	22.0	0.0	0.0	0.0		------	-------
TOTAL						854.38		852.58	1712.36

ESTIMATE USING TRAPEZOID RULE	ESTIMATE USING SIMPSON'S RULE
Volume = Interval x Total Col(7) = (1.0) (854.38) = 854.38 CU.FT	Volume = $\frac{\text{Interval}}{3}$ x (Total Col (8) + Total Col (9)) = $\frac{(1)}{(3)}$ (852.58 + 1712.36) = 854.98 CU.FT.
Tonnage = Volume x Bulk Density x $\frac{(1)}{(2000)}$ = (854.38)(47.19)$\frac{(1)}{(2000)}$ = 20.16 TONS	Tonnage = Volume x Bulk Density x $\frac{(1)}{(2000)}$ = (854.98)(47.19)$\frac{(1)}{(2000)}$ = 20.16 TONS